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ABSTRACT

This document is one of three made up of newsletters from the School Mathematics Study Group's (SMSG) series of newsletters written between 1959 and 1972. This set contains newsletters 2, 3, 10, and 15, reporting the evaluations of student achievement, of experimental units, and of texts; newsletters 5, 12, 15, and 19, discussing teacher inservice activities; newsletters 30 and 35, which report the status of curriculum projects and the ELMA and NLSMA studies; and newsletter 39, which describes canonical instructional procedures and makes suggestions for research programs. Related documents are SE 017 485 and SE 017 487. (JP)

SCHOOL MATHEMATICS STUDY GROUP

Newsletter No. 2

June 1959

Financial support of the
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has been provided by the
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CONTENTS

THE MINNESOTA NATIONAL LABORATORY FOR THE IMPROVEMENT OF SECONDARY MATHEMATICS

1. *History*

At the conference in Cambridge, February 28th to March 1, 1958, which led to the formation of the School Mathematics Study Group a resolution was passed unanimously "welcoming action by the State of Minnesota to provide facilities for statewide testing of the materials to be produced by the SMSG." Minnesota was singled out for this attention because favorable conditions seemed to exist there for obtaining the necessary coöperation.

Discussion was initiated with John Bystrom, president of the State Board of Education, Dean M. Schweickhard, Commissioner of Education, Farley D. Bright, Assistant Commissioner of Education, and other members of the staff of the State Department of Education. It was proposed that an agency for educational experimentation be established as part of the State Department of Education with a small central staff to be supported by the state, consisting of a professional mathematician, an assistant, and a secretary. This agency was to do experimentation on a contractual basis for SMSG.

Professor Begle visited Minnesota in May and met with the various people who would be involved in such experimentation including mathematicians, local and state educational administrators, teacher educators, statisticians, psychologists, and teachers. He recommended to his advisory committee that the Minnesota proposal be adopted on the basis of the "enthusiasm and evident coöperation from all segments of the educational community."

On July 22d, the State Board of Education formally approved the plan. The Louis W. and Maud Hill Family Foundation in St. Paul made a grant of $11,000 to the State Department of Education on August 11th to enable this Laboratory to begin operation on September 1st. Professor P. C. Rosenbloom of the Department of Mathematics, Institute of Technology, University of Minnesota was appointed director.

Among the duties of the director specified in his contract are to "maintain liaison with the national office of the School Mathematics Study Group at Yale University," and to "be responsible for planning programs for the utilization of money grants from the Mathematics Study Group and submit the same to the State Board of Education." These provisions assure that this Laboratory will maintain a close permanent working relationship with the SMSG.

1a. *Legislation*

On April 24, 1959, the Governor of Minnesota signed into law an act permitting Minnesota to accept the provisions and benefits of Public Law 85–864 (The National Defense Education Act of 1958). The State Plan for Title III of this act provides that Federal funds may be used to pay 50% of the cost of personnel and operations of the state office of the Mathematics Laboratory after June 30, 1959.

The use of state funds to pay the other 50% of the Mathematics Laboratory state office personnel and operation costs must be approved by the Minnesota Legislature which is now in session. Both the House and Senate committees on State Department finances and appropriations have acted favorably but final legislative approval has not yet been granted.

2. *Organization and Operations*

This Laboratory is part of the Division of Instruction, headed by Farley D. Bright in the Minnesota State Department of Education. The staff consists of P. C. Rosenbloom, Director, a secretary, and an assistant to be appointed. It has an advisory committee consisting of:

Professor Roland C. Anderson
St. Cloud State College
St. Cloud, Minnesota

Farley D. Bright
Assistant State Commissioner of Education
St. Paul, Minnesota

Stanley Hill, Vice-President
Minnesota Mutual Life Insurance Co.
St. Paul, Minnesota

Roy Isackson, Principal
Como Junior High School
St. Paul, Minnesota

Harvey O. Jackson
Curriculum Consultant Minneapolis Public Schools
Minneapolis, Minnesota

Professor Palmer O. Johnson
University of Minnesota
Minneapolis, Minnesota

Professor Gerhard K. Kalisch
SLA, University of Minnesota
Minneapolis, Minnesota

Nolan C. Kearney
Assistant Superintendent of Schools
St. Paul, Minnesota

Wilber L. Layton, Assistant Director
Student Counseling Bureau
University of Minnesota
Minneapolis, Minnesota

Professor William R. McEwen
Science and Mathematics
Duluth Branch, University of Minnesota
Duluth, Minnesota

Professor Kenneth O. May
Carleton College
Northfield, Minnesota

George Membrez
Educational Consultant
Minnesota Mining and Manufacturing Co.
St. Paul, Minnesota

Professor Samuel H. Popper
College of Education
University of Minnesota
Minneapolis, Minnesota

Dr. Roy Prentis
State College Board
MEA Building
St. Paul, Minnesota

Walter W. Richardson
Superintendent of Schools
North St. Paul, Minnesota

Sister Seraphim, Professor of Mathematics
College of St. Catherine
St. Paul, Minnesota

Roger Thompson, President
Minnesota Council of Mathematics Teachers
St. Louis Park High School
St. Louis Park, Minnesota

Professor P. O. Johnson is the consultant on statistics, and Professor E. O. Swanson of the Student Counseling Bureau, University of Minnesota, is the consultant on testing.

On September 1st, a letter was sent out over the signature of Commissioner Schweickhard to every superintendent in the state explaining the program of the SMSG and this Laboratory, and including applications for participation by school and by teacher. Later such a letter was also sent to the head of every private school in the state. Participation by school and by teacher is entirely voluntary.

At the time that selections were made for this year's experiments, the Laboratory had received 118 applications from teachers in 88 schools throughout the state. Teachers were selected from the population of applicants, stratified by population of the community, on the basis of their professional qualifications as measured by experience, undergraduate and graduate courses in mathematics, membership in professional mathematical organizations, and activities for the advancement of mathematics and science teaching. Twenty-one teachers were selected for the 7th and 8th grade experiment and 15 for the 9th grade experiment.

At present the Laboratory has 156 applications from teachers in 110 schools.

3. *Purposes*

This Laboratory has the function of performing scientific experimentation and evaluation for the SMSG. Its purpose is not to recommend materials for adoption. With each experiment the Laboratory will publish a detailed technical report, giving full information on procedures and results sufficient for checking by independent investigators, on the basis of which school people can make their own decisions. This information will also be useful to the writing teams of the SMSG for revision of the materials.

4. *Experiments* 1958–59

This year the Laboratory is conducting two experiments for the SMSG, one in 7th and 8th grades and one in 9th grade.

In the 7th and 8th grades the Laboratory is attempting to give a preliminary evaluation of the fourteen units

produced by the SMSG last summer. No comparison is being made with the conventional course at present. The Laboratory wishes to investigate such questions as how long it takes to cover each unit, how well pupils of given ability can master it, what aid a teacher needs to handle the new material, what is the proper grade placement of each grade topic, etc. Other problems under consideration are the validity of our measures of teacher qualification, interest and attitudes evoked by the materials, vocabulary and style, item analysis of the unit tests, etc. Rough spots in the students' and teachers' materials and the tests will be searched for. Conventional achievement tests will also be given in June and September.

Not all of these questions can be satisfactorily studied this year. The time pressure of organizing everything at once in September made it difficult to arrange matters as one would have wished. It is hoped that next year the work will proceed much more smoothly.

Of special interest may be that among the experimental classes are an average 6th grade class and a slow 9th grade class.

In the 9th grade experiment a comparison is being made between the materials produced by the University of Illinois Committee on School Mathematics, directed by Professor Max Beberman, and a conventional 9th grade course. There are four experimental classes using the Illinois materials, taught by teachers trained at an academic year seminar at Carleton College during 1957–58. Each of these teachers is also teaching a control class. In two cases both experimental and control classes consist of average 9th graders who enrolled in algebra. In one case both classes consist of (5) superior 8th graders. In all of these cases there was a random choice of the class assigned to the Illinois materials.

These four teachers are among the most highly qualified people teaching 9th grade in the state. In addition there are eleven other control classes in separated schools taught by teachers approximately as well qualified as the experimental teachers. One of the control teachers has also been trained with the Illinois material.

Comparisons will be made both with standard achievement tests and with specially devised tests. Interest and attitudes are also being studied.

The UICSM has coöperated fully in this experiment.

5. *Plans for the Future*

Next year the Laboratory expects to have about 20 experimental classes per grade in grades 7 to 12 using the SMSG materials. In some cases the experiment will be more of a pilot venture. Some three-way comparisons with the Illinois and the conventional materials may be attempted. Various questions concerning articulation between experimental and conventional courses may be studied. The feasibility of using the experimental materials in preservice education will also be studied.

During the summer of 1959, workshops on the SMSG materials will be held at 19 of the 28 colleges and junior colleges in Minnesota. This program is sponsored by the Laboratory and will be financed by the National Science Foundation. No stipends will be offered to the teachers, so that the workshops will be open to all who wish to attend. The Minnesota School Board Association will recommend to its members to seek means to make it financially possible for at least one teacher from each district to attend. The purpose of these workshops is to make as many teachers as possible familiar with the new materials. Attendance at a workshop will be neither a necessary nor a sufficient condition for participation in the program of the Laboratory next year. There is no implication that the SMSG materials are recommended for adoption.

The Laboratory, in coöperation with the Correspondence Study Division of the University of Minnesota, will offer correspondence courses with the SMSG materials on the appropriate grade level to the top one half percent of the secondary school pupils in the state. The Student Counseling Bureau will assist in the identification of these students.

The Laboratory also hopes to offer a correspondence course on the foundations of mathematics and on the SMSG materials to every member of the Minnesota Council of Mathematics Teachers.

To the extent that its budget and manpower will permit, the Minnesota National Laboratory will make its correspondence courses available to people in other states.

6. *The Minnesota School Mathematics Center*

The Minnesota School Mathematics Center was also started last August with the help of a grant of $14,000 from the Hill Family Foundation. Since October 1st its activities have also been supported by the SMSG. It is independent of the Minnesota National Laboratory but is coördinated with it by having a common director.

The function of the Center is to support creative joint work on school mathematics by school and college teachers. At present it has a 7th and 8th grade writing team consisting of five teachers working with Professor Donovan O. Johnson of the College of Education, University of Minnesota, and Professor P. C. Rosenbloom, a 9th grade writing team consisting of two high school teachers working with Professor F. G. Koehler of the Institute of Technology, University of Minnesota and Professor O. E. Stanaitis of St. Olaf College. It also has an elementary teacher working with Professor Rosenbloom on grades 5 and 6. This latter work is supported by the Hill Family Foundation.

The elementary teacher and the members of the 7th and 8th grade writing team are also trying the SMSG experimental units in their classes.

It is hoped that the Center will be one of the projects supported by the Industry-Education Board of the Minnesota Academy of Sciences. This Board was formed last November to coördinate industry aid to education in Minnesota.

7. *Conclusion*

The Minnesota Council of Mathematics Teachers has devoted a good share of its programs and its newsletters to the experimentation with new curricular materials. The Minnesota Association of Secondary School Principals and the Minnesota School Board Association have recommended to their members full coöperation with the work of the Laboratory. The Minnesota Association of School Administrators has also supported the activities of the Laboratory. The interest and enthusiasm of all these groups of schoolmen have been most gratifying.

Many members of the staff of the Minnesota State Department of Education have contributed time and energy to the success of the Laboratory. The program owes much to the generous spirit of coöperation shown by all concerned.

For further information, inquiries may be made of:

Minnesota National Laboratory for the Improvement of Secondary School Mathematics
Department of Education
301 State Office Building
St. Paul, Minnesota

or

Minnesota School Mathematics Center
Institute of Technology
Department of Mathematics
University of Minnesota
Minneapolis 14, Minnesota

MINNESOTA'S ATTACK ON THE PROBLEMS OF EVALUATION

BY

PAUL C. ROSENBLOOM
University of Minnesota

AND

MINNESOTA STATE DEPARTMENT OF EDUCATION

The evaluation of the materials produced by the School Mathematics Study Group is a research problem of a magnitude rarely attempted before in education, and will require for its success a team composed of experts with competence in a number of different fields.

The outcomes of the experiments are random variables, and so are the variables on which they depend. In order to separate out the effects of the material from those of teacher and pupil ability, grouping or lack of it, etc., it will be necessary to apply the modern methods of statistical analysis and design of experiment, developed during the past 30 years by Fisher, Wald, and others. We are fortunate to have in our work in Minnesota the active collaboration of Professor Palmer O. Johnson to handle the statistical problems.

We must have appropriate measuring instruments. For this we must have the collaboration of a professional psychologist and a professional mathematician. Professor Edward O. Swanson and I shall be working together on these problems.

In the past proposals for curricular change have usually been evaluated by means of the standard achievement tests. These tests measure almost exclusively routine mastery of computational skills. The only part of such tests which comes near to measuring mathematical ability is the one labeled "arithmetical reasoning," where the translation of problems into mathematical language goes beyond mechanical computation. The so-called tests of "concepts" turn out to be mostly tests of vocabulary. The tests are biased in favor of the standard curriculum. Judging from internal evidence, I would say that these tests are the product of a collaboration between first-rate psychologists and less than first-rate mathematicians.

Of all the standard tests which I have examined, the one which comes closest to our needs is the Sequential

Tests of Educational Progress (STEP) battery. These are not yet satisfactorily normed, and omit many important aspects of mathematical thinking. We shall, therefore, supplement the STEP battery by tests of our own devising.

We must correlate a pupil's achievement with his native ability and try to distinguish the effects of the new treatment from those of his own intelligence. Intelligence is apparently a vector quantity with a certain structure of correlation between its components. Although the exact structure is still a matter of controversy among the followers of Spearman and Thurstone, still the pattern which emerges from recent expositions by Vernon, Guilford, and Guttman shows remarkable agreement on essential features. We have, therefore, decided to use the Differential Aptitude Tests for the measurement of intelligence. Guilford announced the development of more refined instruments, but these do not yet seem to be generally available. We may expect to find that a child's score on certain parts of the O.A.T. will have a higher correlation than others with his achievement on the mathematical tests we use.

For example, with our present techniques of communicating with the child, his performance of a real test of understanding of concepts may be almost as dependent on his verbal ability as on his mathematical ability.

A test of understanding must differentiate between such levels as recognizing a special case of a general principle, stating the principle in words, stating principle in symbols, applying the principle directly in a one-step problem, and applying it in a longer chain of reasoning. In order to devise such a test for use on a mass basis, we must combine a careful analysis of the mathematical content with the modern techniques for test construction and validation.

We should like to test ability to generalize. I have a good idea of what I want to get at, but Professor Swanson and I will have to work hard to adapt mass testing techniques to this problem. I should like to know, for example, how many terms of the series $1 + 3 + 5 + 7 + \ldots$ a child has to add before he sees the law that the sum of the first n odd numbers is n^2.

Suppose we teach a child the following game played by two people with toothpicks. Any number of toothpicks are placed in a pile. The players agree on the maximum number which may be taken in each turn. Each player in his turn takes at least one toothpick, but no more than the agreed maximum. The one who takes the last toothpick wins. The child can exhibit various stages of understanding the general strategy. If the agreed maximum is 3, then he may observe that he can always win if he leaves 4 toothpicks. He may then discover that 8 is a winning position. He may later see that multiples of 4 are the winning positions. Finally, he may generalize to the principle that if n is the maximum which can be taken each time, then the winning positions are the multiples of $n + 1$.

This ability to generalize overlaps, but does not coincide with, the power to abstract and to use symbols. A child may recognize $3 \cdot (4 + 5) = (3 \cdot 4) + (3 \cdot 5)$ as a special case of the law $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$. He may be able to apply the law to another case $2 \cdot (3 + 4) = (2 \cdot 3) + (2 \cdot 4)$ but not to the problem of multiplying out $(1 + n)^2$.

Very little has been done to measure creativity, although Thurstone's verbal fluency test and some tests which ask for suggestions for inventions are a start in this direction. The ability we wish to assess here is that of asking good questions, recognizing problems, and inventing new approaches. For example, one of my 5th graders after a couple of months of my teaching, came to me with the question, "I have been making a table of the multiples of 11, and noticed that $11 \times 25 = 275$ and $2 + 5 = 7$, $11 \times 31 = 341$ and $3 + 1 = 4$, but that $11 \times 38 = 418$ but $4 + 8 = 1$. Is there a law?" It will be difficult to test the ability to discover such relationships independently without giving time for meditation.

No one with real competence in mathematics would be surprised at Thorndike's negative results on transfer of training. For Thorndike did not take into account whether the mathematics was taught by rote or for understanding. On the other hand, Brownell found, when he taught subtraction of 2 digit numbers and then tested the pupils with problems on subtraction of 3 digit numbers, that he obtained better results when he taught the meaning of the process than when he taught a mechanical skill.

A fundamental question about our new materials is how much do they improve a child's ability to solve a problem for which he has not been specifically trained. Does a child do better on such a problem as the one of the three men and five hats after he has studied Unit I than before? Anyone who habitually gives non-routine problems in tests knows how they separate the students who conscientiously do all their homework from the ones who think about their work.

When we measure the effect of the SMSG units on the above-mentioned abilities, we are finding out how well we are conveying to the student the mathematical content of the course. But this is not enough. In the writing conference we tried to embody in our work that mathematics is so intrinsically fascinating "that only the most extravagant incompetence could make it seem dull." Did we succeed in arousing the interest of the children? Even if no significant differences between the new and the standard course are found in other respects, if one stimulates more children to do mathematics, then it is a better course.

How can we measure interest? Of course, a crude approach is to ask direct questions. A really good questionnaire is more subtle and therefore more difficult to construct. Still, surprisingly good results have been obtained in the consumer preference studies at the University of Michigan and in Guttman's work on the intentions of Israeli soldiers.

Even if we could devise a good questionnaire, we would still have to validate it by some correlation with overt behavior. We shall try to measure the extent to which the course stimulates reading library books on mathematics. We plan also to prepare short enrichment brochures related to topics covered in the units and to supply them to individual pupils on request. We shall make available to the children additional problems and materials for projects. We shall take a sampling of children's diaries to find out to what extent we have stimulated them to extracurricular activities in mathematics.

In 7th and 8th grade we are dealing with all pupils including a majority who will not use mathematics vocationally. In a society which depends increasingly on basic research in mathematics and science, we must develop attitudes and appreciations which will ensure appropriate public support. This is part of our contribution to the liberal education of every citizen.

Some beginnings have been made on measuring attitudes and appreciations in more mature students. The Kuder preference test may be adaptable to younger students, but at present depend too much on a pupil's knowledge of the world of work. *Fortune Magazine* has published two interesting surveys, one on which school subjects students like or dislike the most, and one on students' mental picture of the scientist. How will the biographical material and information on the work of the mathematician included in our units affect the children's concept of a mathematician? I call your attention to the concerted effort in the Soviet Union to provide children with biographical materials on "heroes of culture," written by leading scholars. Have we succeeded in conveying to the children some appreciation of beauty and elegance in mathematics, and some understanding of the value of mathematics to our society and its place in our culture?

We are conceiving our program of evaluation as the work of a research team, including mathematicians, statisticians, psychologists, teachers, and students, and we make a serious attempt to help the teachers and children view themselves as investigators, not as guinea pigs. We have already learned several things from our experimental teachers. One of them, Charles Bastis, gave the children a questionnaire after teaching Unit I, and asked, among other things, how much time the pupil spent per week discussing school work with his parents. A preliminary study showed a higher correlation between this item and the level of achievement than between IQ and achievement. If this finding is substantiated, then we must measure parental interest. Furthermore, for better results we must take deliberate steps in our writing and teaching to involve the parents.

Related to this is the effect of the materials on the teacher. A student teacher working under Marshall Kaner had much greater success in teaching decimals after observing him teach Unit II than she had had before. We hope to be able to introduce these materials in teacher education and to follow up the teachers after they graduate. We also want to know how teaching our materials affects a teacher's performance in other classes.

Clearly a research program of such scope must be long range. The materials are only in a preliminary form, and we must first answer many questions regarding their characteristics. We need to know how long it takes to cover each unit, how well children of given ability can master each unit, which topics are suitable for which grade, what should be the sequence of topics, what aids the teacher needs in teaching each unit, etc.

We hope to obtain some of the answers this year. The writing team will use these results as a basis for revising the units next summer, and next year we hope to pit the new course against the traditional courses.

The research we have described is part of the program of the Minnesota National Laboratory for the Improvement of Secondary School Mathematics. This Laboratory, and the Minnesota School Mathematics Center, are described in the preceding article. We expect that both the Laboratory and the Center will be permanent institutions. If the Laboratory should prove successful, the State Board of Education contemplates extending it to all other fields as the corresponding national projects for curricular revision are organized. We hope in this way to establish research in curriculum as a normal and essential activity of the State Department of Education. We hope that our experimentation program in mathematics will serve as a pattern for other states and for other subjects. In this way we may contribute to a general improvement in the quality of education in the United States.

EXPERIMENTAL UNITS FOR GRADES 7 AND 8

We have received numerous inquiries concerning the classroom use of SMSG text materials in the coming academic year. All SMSG text materials are new and experimental and their classroom use will probably require a certain amount of additional training on the part of the teacher. It is therefore SMSG policy to discourage classroom use of these text materials until they have been tried out in the Centers which we have established for this purpose. These Centers consist of experienced teachers together with a college or university mathematician to provide whatever inservice training is needed. Periodic reports from these Centers give us information on the teachability of the material and the ability level needed for student mastery.

In general, all our text materials will be rewritten at least once on the basis of the experience thus gained through our Centers.

The only SMSG text materials which have been through such a tryout are the Experimental Units for Grades 7 and 8. Preliminary reports on these from our Centers are quite favorable and we have therefore decided to make these generally available for classroom use for the academic year 1959–60.

It should be remembered that these units do not form a complete course. They can be used either as supplements to the regular course, for either of these grades, or as replacements for certain chapters of the regular textbook.

In the next issue of this newsletter, which will appear late in the summer, there will be a digest of the reports on each of these units which we have received from our Centers. This information, together with the units themselves, which are still available (see page 23), should make it possible for a teacher to make a decision on the use of any particular unit.

The units will be revised slightly during the summer, mainly to eliminate typographical errors. They will be made available in three separate booklets, rather than the one volume which is now available. The first of these booklets will contain those units devoted to the structure of number systems. The second will contain those units concerned with geometry. The third will contain those units devoted to the applications of mathematics.

The price of these units will be comparable to that of the current version. Instructions for ordering will be in the next issue of this newsletter. Please do NOT order now.

A GEOMETRY COURSE FOR HIGH SCHOOL TEACHERS

A second volume in the SMSG series *Studies in Mathematics* is now available. This series is meant primarily for high school teachers and is devoted to topics directly related to high school mathematics courses. Particular attention is paid to those topics which play an important part in the courses being developed by SMSG. Volume 2 of this series, entitled "Euclidean Geometry Based on Ruler and Protractor Axioms," is intended to provide familiarity with the approach to 10th grade Euclidean geometry which has been adopted by SMSG. It is in the form of a textbook for high school teachers who are already familiar with Euclidean geometry. The course is concerned primarily with the beginnings of the subject, where the SMSG course will differ considerably from the standard high school geometry course.

This was written primarily for use in Summer Institutes in 1959 and should be considered a preliminary version of the course. It is planned to revise this course during the academic year 1959–60, taking into account comments, criticisms, and suggestions which may result from its use during the coming summer.

PUBLICATIONS AVAILABLE

TO ORDER, USE THE FORM ON THE INSIDE BACK COVER AND SEND TO School Mathematics Study Group
Box 2029 Yale Station
New Haven, Connecticut

STUDY GUIDE IN MODERN ALGEBRA

The purpose of this Study Guide is to provide assistance to teachers who wish to improve their professional competence in modern algebra by study either individually or in small groups. It contains a list of basic topics and ideas which should be part of the mathematical equipment of teachers of introductory (9th grade) algebra. There is a short bibliography and, for each of the basic topics and ideas, detailed references to this bibliography.

FREE

SOME BASIC MATHEMATICAL CONCEPTS, by R. D. Luce

An exposition of elementary set theory, together with illustrations of the use of set concepts in various parts of mathematics.

Price: $1.00

EUCLIDEAN GEOMETRY BASED ON RULER AND PROTRACTOR AXIOMS (preliminary edition) by C. W. Curtis, P. H. Daus, and R. J. Walker

A study, written for high school teachers, of the approach to Euclidean Geometry which will be used in the SMSG 10th grade textbook.

Price: $1.00

EXPERIMENTAL UNITS FOR GRADES 7 AND 8

Fourteen experimental units for these grades, together with a teacher's guide for each unit and answers to the problems.

Price: $2.00

An Apology.

Some of you ordered SMSG publications and received them only after a long delay. This was due to an underestimate on our part of the demand for these materials and to a delay in reprinting because of the printer's crowded schedule.

We apologize for this delay. We will do our best to see that such delays do not occur in the future.

If you have ordered SMSG publications and have not received them by the time this NEWSLETTER reaches you, please notify us at once, giving full details of your order. *Be sure to include your address.* A few orders are being held at SMSG headquarters because no address was included.

If you are not now on our mailing list but wish to receive further issues of this NEWSLETTER, please request, by means of a post card, that your name be added to the mailing list.

ORDER FORM

____copies of STUDY GUIDE IN MODERN ALGEBRA — FREE

____copies of SOME BASIC MATHEMATICAL CONCEPTS — $1.00 each $_____

____copies EUCLIDEAN GEOMETRY BASED ON RULER AND PROTRACTOR AXIOMS (preliminary edition) — $1.00 each $_____

____copies EXPERIMENTAL UNITS FOR GRADES SEVEN AND EIGHT — $2.00 per complete set $_____

Total enclosed $_____

Address to which above order should be sent.

____

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Remittance must accompany each order. Please make checks payable to Yale University, and mail to

SCHOOL MATHEMATICS STUDY GROUP
Box 2029 Yale Station
New Haven, Connecticut

SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 3

September 1959

CONTENTS

Financial Support of the School Mathematics Study Group has been provided by the National Science Foundation

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TEACHERS' REPORTS ON EXPERIMENTAL UNITS FOR GRADES 7 AND 8

As was promised in NEWSLETTER No. 2, we present here digests of the reports from teachers who used these units in their classes during the past academic year. The purpose of this report is to make it easier for teachers to decide whether or not they wish to use these units in their classes during the coming academic year.

It should be remembered that these units do not form a complete course. They can be used either as supplements to the regular course or as replacements for certain chapters of the regular textbooks for these grades.

Each teacher in the SMSG Centers was asked to fill out a brief questionnaire on each unit taught. A copy of this questionnaire appears on Page 4. The digests of teacher comments were prepared from the completed questionnaires.

It should be noted that some of the units were taught by only a few teachers, so that information on them is less reliable than for others.

These units are now available in three volumes. The first volume is devoted to the structure of number systems, the second to geometry, and the third to applications. For the benefit of those who have seen these units before, the original numbering of the units has been preserved.

The digests of teacher comments are arranged according to the order in which the units appear in these three volumes.

Date....................

UNIT:..

Name of Teacher:..................................

Name of School:...................................

City:................State:..................

Number of days given to the teaching

(including testing) of this unit:.....................

Approximate dates:................................

USE THE BACK OF THIS SHEET IF YOU NEED EXTRA SPACE TO ANSWER ANY OF THE QUESTIONS BELOW

1. Make a statement about the ability level of the pupils in the class and state whether your school uses some plan of homogeneous grouping.

2. What parts of the unit proved to be the most teachable?

3. What parts of the unit proved to be the most difficult to teach?
   Did you omit any part?........

4. Did you use any supplementary developmental materials?........
   If so, what were they, and at what points were they used?

5. Did you find it necessary to provide the pupils with additional practice material?........
   If so, was it from textbooks or did you write your own?

6. Do you think that a unit on this topic should be included in regular textbooks for 7th and 8th grades?
   ....

7. Please make ANY additional comments about your teaching experience with this unit which you think would be helpful to the Panel responsible for preparing and experimenting with textbook materials for grades 7 and 8.

VOLUME I

NUMBER SYSTEMS

UNIT II—NUMERATION

Table of Contents:

Summary of Teachers' Comments

Seventy-five teachers reported that they had taught this unit in 110 classes. Fifty of the classes were grouped according to ability (35 high, 9 medium, 6 low) and 60 were heterogeneously grouped. Most of the classes were in the seventh grade with the remainder in the eighth grade and a few in the sixth, ninth, or tenth grades. Opinion was close to unanimous that material in the spirit of this unit should be included in a seventh grade course.

The time spent on the unit varied from 10 to 27 days with an average of about 16 days. It is suggested that 15–18 days is a reasonable amount of time for the unit. There was some flagging of pupil interest when the unit was continued too long. Presumably, when teachers have become thoroughly familiar with the material and appropriate teaching techniques, a considerable reduction in time may be effected.

It is recommended that the section on divisibility be omitted or treated lightly except in the most capable classes. Teachers reported on the sections as follows:

Most teachable:	(1.) Other bases
	(2.) Historical treatment
	(3.) Operations
Most difficult:	(1.) Tests for divisibility
	(2.) Base seven
	(3.) Changing bases

A few representative comments of teachers follow:

I have never seen interest as high toward mathematics. There was no need for motivation.

I feel that more exercises should be included. The children need lots of practice before they can handle this material with understanding.

I noticed the slower pupils expressed great interest at first but lost interest to some extent in the more difficult portions of the unit.

I am sure now that the time it takes to teach it depends somewhat on how much experience one has in teaching this material.

One parent commented that the advent of the new mathematics had made their dinner conversation very stimulating. She hoped the "new" math was going to continue next year.

On the whole the unit was well received. I have a few pupils who were very poor producers, but did very fine work on this unit. On the other hand, a few pupils (2) did not make any effort to absorb this new material.

It was easier for me to teach this unit to the seventh graders than it had been previously to try to teach some of these same ideas to adults with mind sets of various kinds.

Base two was a novelty and caused much fun in the class.

UNIT III—NATURAL NUMBERS AND ZERO

Table of Contents:

Summary of Teachers' Comments

Reports from 76 teachers indicated that the time used on this unit varied from 5 to 30 days and that the pupils varied in ability. There seemed to be equal numbers of high, medium, and heterogeneous classes involved.

Tabulated results showing the topics regarded as easiest or most difficult to teach follow:

Topic	*Easiest to Teach*	*Most difficult to Teach*
Closure	2	43
Commutative ands asociative	34	5
Distributive	0	32
Inverse	15	1
Betweenness	16	8
1	12	0
0	10	7
Division by zero	0	5
Inequalities	2	0
All	27	0
Counting	8	0
None	0	23

Two teachers reported that they had not taught all the topics. There is general agreement among the teachers who reported that Unit III is "... the most interesting and useful unit studied thus far"; "... one of the best units that I have ever taught." Most reports reflect the feeling that the closer examination of the familiar counting numbers and the equally familiar operations are not only important, but interesting to the student. Equally important is the rather gentle introduction to the use of letter symbols for numbers which it affords. A large number of classes omitted no topics and did not see the need for supplementary materials except in the sections mentioned below. Most felt that this unit is desirable for the seventh grade; a small number would rather have it in the eighth grade.

More specific comments which occurred in significant numbers of reports follow:

1. An overwhelming majority of teachers reported that the section which introduces the concept of closure was the most difficult to teach. There was general feeling that classroom discussion and drill should be expanded beyond that indicated in the unit. In connection with this a little time might well be devoted to a discussion of the intuitive idea of "sets."

2. A sizable number of teachers indicated that students had difficulty with the distributive property. This section was frequently mentioned as one in which extra drill exercise might well be assigned.

3. While comparatively few teachers reported that the section on the number zero was especially difficult, a large number of them indicated that such things as division by zero and the conventional exclusion of zero from the set of counting numbers (even though zero "may be used to count" in a special sense) required additional emphasis and explanation.

Additional Materials Used

Practice on principle	Formulas from geometry
Information on closure	Abacus binomials
Cardboard squares and circles for grouping	Lateral binomials
Number scale	Trapezoid formula
Simple interest problems	Drill on inverse operations

UNIT IV—FACTORING AND PRIMES

Table of Contents:

Summary of Teachers' Comments

There were 67 teacher reports from centers which included classes of pupils of all levels of ability. The number of teaching days spent on the unit ranged from 4 to 28 with a median of 12 days.

Two questions on the report form were: (1) What topics were the easiest to teach? And (2) What topics were the most difficult to teach? The topics listed by the teachers with the tabulated results follow:

Topic	*Easy*	*Difficult*
Primes	20	2
Factorization	24	1
G. C. F.	8	16
L. C. M.	9	23
Sieve of Eratosthenes	6	4
Deductive Proof	0	4
Vocabulary	0	2
Odds and Evens	1	1
(other bases)		
Composites	2	0
Multiples	4	0
The entire unit	10	0

Four reports did not give an answer to either question.

In general, the reports indicated that this unit was teachable, easily understood, and rather popular with the pupils. The unit was considered as a good motivating vehicle. Many questions were raised by the pupils. The pupils realized that the use of their imagination was important. Some pupils became interested in history and read biographies of famous mathematicians. Many enjoyed understanding processes previously done without understanding.

It was felt that this unit helped develop number consciousness, and also was an excellent preparation for algebra. The indications were that this unit definitely should be taught in the seventh grade. The type of proof used was new to the pupils and required much work in order to overcome the beginning difficulties. The vocabulary presented difficulties to the average pupils and the slow learner. The sections on G. C. F. and the L. C.

M. caused the most trouble, with the L. C. M. taking the lead. It was felt that the L. C. M. followed too closely on the G. C. F. Some teachers felt that there should be more problems directed toward the talented group of pupils.

Listed below are direct quotations from some of the reports:

We enjoyed this unit and considered it to be another approach to our former knowledge of fractions. In adding and subtracting fractions we now use L. C. M. information and in multiplication and division the new knowledge of G. C. F. The value of these newer techniques cannot be stressed too strongly for their great insight into similar algebraic expressions of the ninth grade. The pupils enjoyed using other bases and deciding on odds and evens. I was happy to see that they could recall the other bases quite efficiently.

I noticed that this unit developed a sort of "number consciousness" that the pupils did not have before. They seemed to look at numbers a little differently and to associate them with such ideas as odd, even, prime, composite, even + even, even + one, odd + one, with or without the factor two.

One of the most difficult parts of the teaching at first was to help pupils develop skill in using informal deduction to show that what they had suspected (by induction) was true. Often we had done some of this in class and I had illustrated some of the deductive arguments they could use, the pupils improved and would try to use some of the background they had built in studying Unit III.

The pupils' reaction to Unit IV was negative in the beginning. It was difficult to stimulate their interest and make them realize the value of this material. As we progressed, however, individuals began to see that what they had previously done mechanically, now had meaning which they understood.

The teacher who made the above statement also listed some comments from the pupils as follows:

(a) We are learning this too late.
(b) It's a longer, but surer way to work.
(c) I was very glad I had studied Unit IV before I took the Iowa Reading Test.
(d) This kind of study gives us a fine background, so that we can talk more intelligently about numbers.

Some pupils brought in supplementary materials and all were extremely interested in the work of the unit. We had reports based on an article from the June, 1958 *Fortune* and an article from the December, 1958 *Scientific American*. In retrospect, I believe that I should have given more work involving computations with fractions, applying the concepts of G. C. F. and L. C. M. There was some tendency to confuse the two ideas.

The section of this unit which produced the most interesting discussion was the process of reasoning inductively—then deductively about the even-odd relationships. Some of the deductive proofs presented by the pupils were not conclusive, but the germ of the method was present. Several pupils were so interested I gave them some "number theory" problems to prove.

UNIT IV-A—SUPPLEMENTARY TESTS FOR DIVISIBILITY AND REPEATING DECIMALS

Table of Contents:

One teacher reported her experience teaching this unit to a class described as "very high—accidental gifted grouping." This teacher reported that the sections on divisibility and why a decimal repeats were easy to teach, and that the general form of proof was the most difficult part to teach. She recommends that the unit definitely be taught to eighth graders as preparation for algebra. The report also indicated that some of the children made some very interesting discoveries of their own.

UNIT V—THE NON-NEGATIVE RATIONAL NUMBERS

Table of Contents:

Summary of Teachers' Comments

The number of days given to the unit varied from 7 to 43 with an average of 18. The classes varied in ability to this extent: 19 high level, 6 medium level, and 15 heterogeneously grouped classes. Many classes made use of supplementary developmental and practice materials.

The ease or difficulty of the sections in the opinion of the responding teachers is indicated in the following list:

Topic	*Easiest to Teach*	*Most Difficult to Teach*
Whole numbers and divisibility	5	0
Fractional notation	5	0
Multiplication of rationals	8	2
Equality of rationals	5	1
Division by zero	4	2
Division of rationals	7	5
Addition of rationals	6	5
Summary of properties	2	2
Order of rationals	1	9
Decimal equivalents	6	2
Repeating decimals	6	5
Fractional equivalents of repeating decimals	3	8

Teachers of this unit generally agreed on the importance of most of the material in this unit, but found it slightly harder to maintain high interest than on some of the other units. However, 22 out of 24 felt the material should be introduced.

A majority felt it necessary to use supplementary material, particularly for added practice material. This was the most commonly voiced need. Added materials used by different teachers included the following:

Work on geometric progressions including the "Achilles and the tortoise" paradoxes.
More work on percent from regular texts.
Introduction of negative numbers (because of title of unit).
Ratio and proportion problems.
Practice exercises from corresponding Maryland unit—also use of this unit for developmental work.
Language of sets and simple diagrams to clarify meaning of operations on rationals.
Drill problems on manipulation with rationals.
Formula for changing repeating decimal to equivalent fraction.

No clear conclusions appear possible from the data on ease or difficulty of sections except that the first few sections in general went easily, but that the work on the order relations of the rationals and the work on repeating decimals in general were hard. Several teachers noted, however, that the material on the repeating decimals, though not easy, was particularly stimulating and exciting for the students.

Typical quotations of statements made by teachers follow:

> I feel that this was most valuable in strengthening the pupils' understanding of the various properties. As we work with these units, I find an increasing desire on the part of my students to know reasons for operations. They also show a growing ability to generalize.

> The level of verbalization is very high for seventh and eighth grades in all of these units. Poor readers encounter major difficulties. A number of my better

students have complained that the statements of the exercises were hard to understand.

We had more fun with this unit than any of the others so far. It was also much tougher for most of the students. We spent considerable time making various "proofs." I believe this was most valuable and will carry over into future mathematics work. I am sure the path into algebra will be much smoother for these students than for those in "regular" classes. There is much here to tie arithmetic to algebra and vice versa. I believe that the material in this unit could most effectively be taught at the eighth grade level. With some revision and simplification, I plan to continue to use it with the accelerated seventh grade groups.

UNIT XIV—MATHEMATICAL SYSTEMS

Table of Contents:

Summary of Teachers' Comments

Only 5 teachers reported on the unit. The unit was taught in classes of both average and high ability; all voted that it should be included in the basic course; no material was omitted; the teaching time ranged from 5 to 15 days with an average of about 8 days.

One teacher found the introductory material to Exercises 4 easiest to teach. The other found rectangle changes easiest. The following points were mentioned as being difficult to teach:

1. Algebraic systems without numbers.
2. Motivation for the concepts of closure and inverses.
3. Problem 3 of Exercise 5 needs careful explanation. The following additional information was considered useful: The axes will remain stationary; e.g., the flip about the axis through B in the figure will always be a flip about a vertical axis. This is important when such a flip follows the turning of the triangle about its center.

VOLUME II—GEOMETRY

UNIT VI—NON-METRIC GEOMETRY

Table of Contents:

Summary of Teachers' Comments

The number of days spent on this unit varies from 10 to 22. Most of the pupils seemed to be in the more able group, although some classes were reported as medium or low in ability. Five teachers omitted no part of the unit. A majority felt that the unit included sufficient developmental and practice material. The teachers were almost in complete agreement that the unit should be included in the seventh grade curriculum.

Opinions concerning the individual sections follow:

Topic	*Easiest to Teach*	*Most Difficult to Teach*
Sets and points on a line	10	1
Intersection	5	3
Line segments, half-lines, and rays	11	3
Lines on a point	0	2
One to one correspondence	0	6
Closed curves	4	1
Quadrilateral	0	2
Planes and skew lines	0	14
Special figures	0	18
Whole unit	11	0
No parts of unit	0	6
Angle	0	3

Comments of 18 teachers (23 classes) on Unit VI (Non-Metric Geometry) fall into two patterns as indicated by the following abbreviations: "Understandable and clear"; "a good unit"; "refreshingly different"; "interesting to teach"; "students enjoyed the unit"; "both exciting and frustrating"; "best adapted to better students"; "more developmental material needed"; "some parts too difficult"; "lacked motivation"; "students had considerable difficulty."

Other comments in the nature of advice to other teachers include:

Students and teacher learn together.

Students with I.Q. of 75 learned some of the definitions and participated in the class exercises.

Students who are unfamiliar with the idea of set need a little more work with "sets" prior to the introduction of "intersection sets."

Students may show a preference for traditional material unless the teacher is conscious of his responsibility for maintaining interest initially obtained by the "differentness" of the material.

Do not hurry the class, particularly in the first parts of the unit. Pupils need time to develop concepts.

Interest in the Desargues Configuration may be difficult to obtain.

UNIT VIII—INFORMAL GEOMETRY I

Table of Contents:

Summary of Teachers' Comments

Reports were submitted by 25 teachers. The unit was taught to pupils at all levels but relatively few were described as low level. The number of teaching days varied from 8 to 26.

The following table shows the topics which were reported easiest to teach and the topics which were reported most difficult to teach. The number indicates how many teachers reported these topics.

Topic	*Easiest to Teach*	*Most Difficult to Teach*
Transversal	4	0
Informal proofs	0	5
Principle 3	0	2
Angles of a triangle	3	0
Principles of parallel lines	1	0
Experiment	0	3
Applying principles	1	0
Converses	2	0
Isosceles triangles	2	0

It was apparent that the teachers thought that the vocabulary was difficult at times. Some of them stated that there was too much discovery; for example: cutting out triangles and angles reminded the student of cutting out paper dolls.

One teacher remarked that this unit did not go as well as the early units in her seventh grade class. One stated that for her eighth grade class it was an excellent unit.

Several teachers suggested measuring angles by using a protractor. They proposed that the idea of approximate measurement be introduced at the same time.

It was suggested by several that principles stated in final form should be in the guide for teachers but not in the students' units.

UNIT IX—INFORMAL GEOMETRY II

Table of Contents:

Summary of Teachers' Comments

IX. Seven teachers reported on Unit IX. They gave the number of days spent on the unit as 6, 12, 13, 14, 15, 18, 18, and described the classes as average, high, and heterogeneous.

The following topics were reported as easiest to teach:

Congruent Triangles
Perpendicular Bisectors
Right Triangles
Concurrent Lines

The following topics were reported as most difficult to teach:

Concurrency
Theorem of Pythagoras
Necessary conditions for congruence
Applications of congruence
Converses
Constructing $\sqrt{2}$

Since only 7 teachers reported, the comments are limited. Four thought the unit was difficult, but 2 reported that the unit was "very popular" and 1 classes the unit with VIII as "the two best." Three of the teachers recommended with enthusiasm that the unit be included in the course. Several thought there was too much drawing and measuring and that there were more "discovery" exercises than were needed. Two commented that the "proofs" did not go well in their classes.

UNIT X—MEASUREMEMT AND APPROXIMATION

Table of Contents:

Summary of Teachers' Comments

Unit X was not designed as a first unit on measurement. Apparently this was not clear to all the teachers who responded.

The number of days given to the unit varied from 7 to 15 and the pupils ranged in ability from high to low with most of the classes heterogeneously grouped.

Nine teachers omitted no part of the unit. Five indicated some omissions. About half of the teachers used supplementary developmental materials and a majority found it necessary to provide additional practice material. Only 1 teacher felt that the unit should not be included in either the seventh or eighth grade curriculum.

Reports on the ease or difficulty of the units follow:

Topic	*Easiest to Teach*	*Most Difficult to Teach*
Greatest possible error	10	2
Relative error	3	7
Significant digits	4	6
Precision	12	0
Unit of measure	4	0
None	2	0
Approximate computation	0	2
Approximating error in area	0	12

Comments of teachers on Unit X fall into patterns as indicated by the following typical abbreviations: "Unit not as valuable as some of the others"; "results were frustrating"; "of doubtful value to students with less than average I.Q."; "pupils let down on this unit"; "too much attempted too rapidly"; "too many big words"; "some students were bored—some enjoyed it"; "teacher and some students found the Unit interesting"; "students found Unit interesting"; "wonderful experience"; "this is for eighth grade"; "needs to start at far simpler level"; "introduce actual practice in making linear measurements"; "needs motivation"; "now we have it —what do we do with it?"

The following comments are in the nature of helpful advice to teachers who use the unit:

Use more material at the beginning of the unit to introduce the concepts of measurement and especially the concept that all measurement is approximate.

Good background in decimals and percent is essential; even so students with less than average I.Q. may have difficulty.

Some actual measurements with graduated units will be needed at the beginning. It would seem reasonable to expect seventh graders to be familiar with such measurements—but they are not. Use both English and Metric measure measuring sticks.

VOLUME III—APPLICATIONS

UNIT I—WHAT IS MATHEMATICS AND WHY YOU NEED TO KNOW IT

Summary of Teachers' Comments

Reports on Unit I were received from 75 teachers who taught it in 110 classes. Of these classes 50 were grouped according to ability (35 high, 9 medium, 6 low) and 60 were heterogeneous. Three-fourths of the classes were seventh grade and most of the rest were eighth grade classes. There were a few classes at the sixth, ninth, or tenth grade levels. Most of the teachers agreed that the unit should be included in the seventh grade.

The time spent on the unit varied from 2 to 17 days. Some 70 percent of the teachers taught it from 3 to 8 days. It is suggested that probably 5 or 6 days is a reasonable time for most classes.

The teachers regarded the sections as most easily taught or most difficult in the following order of frequence:

Teachability: (1.) Gauss
(2.) Deduction
(3.) Probability

Most difficult: (1.) Probability
(2.) Deduction
(3.) Gauss

Note that there were differences of opinion with respect to teachability and difficulty.

A few representative comments made by teachers were:

> The interest of students was very high. Also there was participation in discussion by students who normally are reluctant to contribute.
>
> There was very good reaction to this material as an opening unit. There was evidence that the family at home had been brought into some discussions and their interest was whetted for more.
>
> This type of material should be spaced strategically throughout a text.
>
> I recognized that my teaching of the unit to my second participating class was better.
>
> The interest span of the lower group is longer than it was when they were studying traditional work.

UNIT XI—THE SCIENTIFIC SEESAW OR MATHEMATICS AT WORK IN SCIENCE

Table of Contents:

Summary of Teachers' Comments

The chapter on the scientific seesaw was used by 12 teachers in 18 sections, largely with heterogeneous grouping. Those who taught it liked it and invariably found it well received by the students.

Teachers reported spending from 3 to 10 days on the material. At least 5 days were devoted to the chapter in most classes. In several schools the material was taught in coöperation with the science teacher and in at least one instance the unit was taught in the science classroom.

Supplementary material was seldom used although a few teachers introduced other types of levers and a number suggested this as a desirable option. Graphing was hard for some classes and for a few others the law of the lever caused difficulty. In one or two cases the teachers were dissatisfied with the experimental equipment at their disposal. There was general agreement that the chapter was especially valuable as a tie-in with science and as an introduction to applications of mathematics.

UNIT XII—UNCLE SAM AS A STATISTICIAN

Table of Contents:

Summary of Teachers' Comments

Unit XII, "Uncle Sam as a Statistician" was taught to 22 classes by 17 different teachers. Nineteen of the classes contained students of various abilities and in 3 of the classes the students were above average in ability. The time spent on teaching the unit ranged from 6 to 17 days. The average was about 12 days.

In general the teachers found all the units easy to teach. Some teachers indicated that the materials on central tendency and graphs were especially liked by the students. It was reported that some students needed assistance in finding trends from data and the average of deviations. About half of the teachers found it helpful to provide additional supplementary developmental and practice material. Some teachers planned the unit with the social studies teacher and highly recommended such a coöperative arrangement.

With minor suggestions, the teachers who taught the unit recommended its inclusion in the mathematics program for either the seventh or eighth grades.

UNIT XIII—CHANCE

Table of Contents:

Summary of Teachers' Comments

Most of the 22 teachers who reported using the unit spent 6 to 10 days on it, with 7 days being the most representative. One teacher spent as much as 20 days on the unit. The classes were largely of the heterogeneous type. Only 2 teachers had high ability groups.

All teachers seemed to feel that the entire unit was very teachable. In one case, where the material was used with a high ability group, the unit was said to be too easy. It was the feeling of most teachers that no part of the unit was too difficult for the seventh or eighth grade level. Only in one case was vocabulary mentioned as a difficulty. If there were any omissions it was only because of a lack of time rather than for any other reason. In such a case the class omitted an experiment or two.

Ten of the teachers used supplementary practice material. Materials used included:

Original materials by teachers
Newspaper data
Baseball data
Models for chance written by pupils for others to solve.

General comments written by teachers centered around the fact that the unit was interesting to the pupils, fairly easy, and in some cases even too easy. There were some reactions from the pupils regarding the sameness of the exercises which at times made the work monotonous. Some wanted more application to everyday life and others to science.

Some pupils indicated they had to think more than when they used the textbook and preferred the material to that of the textbook. All the teachers agreed that the unit was suitable for seventh and eighth grade levels. One teacher commented that the material would be more suitable if it were broadened.

A number of the teachers showed real enthusiasm for the unit, indicating that pupils "love it," were extremely interested, that it was the "best of all" and that the unit should go further because of their pupils' excitement and enthusiasm for it.

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(Note: Some comments were received too late to be included in the Commentaries for Teachers. They have been taken into account in the digests printed in this NEWSLETTER.)

These volumes can be purchased separately. For those who wish a complete set of units (Commentaries for Teachers included), a reduced price has been set.

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SCHOOL MATHEMATICS STUDY GROUP

Newsletter No. 5

November 1960

CONTENTS

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IN-SERVICE PREPARATION OF MATHEMATICS TEACHERS— EXPERIENCE IN SMSG EXPERIMENTAL CENTERS.

In the academic year 1959–60 over 600 teachers taught from SMSG texts. An unknown but substantial number of teachers used texts produced by other curriculum groups or incorporated some of the new ideas into conventional courses. This year well over 3,000 teachers are using SMSG texts alone. In other words, improved curriculum materials are being used by a significant and rapidly increasing fraction of the mathematics teachers of this country.

However, most of these teachers face a common problem when teaching SMSG texts, or any other improved curriculum materials, for the first time. This is the problem of obtaining the additional preparation in mathematics which these texts require a teacher to possess. Most teachers, through no fault of their own, were not provided in their pre-service preparation with this kind of mathematics. Consequently, most teachers need some assistance either before or during their first use of these texts.

Some teachers will be able to obtain this additional preparation in summer school courses or in Institutes such as those sponsored by the National Science Foundation. However, most teachers will have to obtain this assistance through in-service programs arranged by the school or school system in which the new materials are to be taught. For this reason a summary of SMSG experience in this area may be of interest and value to administrators and teacher groups who are organizing programs to provide additional mathematics preparation for in-service teachers.

During the past academic year SMSG texts were used in 50 Experimental Centers. Each Center consisted of about half a dozen classroom teachers and a subject matter expert who provided the required additional mathematics. In providing this subject matter help for the teachers, SMSG relied almost exclusively on university and college mathematicians. SMSG in this respect benefited from the Physical Science Study Committee. One of the members of this committee pointed out an incidental but very important advantage of this procedure:

"I have said for a long time that if all the PSSC books, films, apparatus and guides were now burned that we still would have had a marked impact on high school physics—and this principally through the contact we have provided for secondary school teachers and university teachers to get together directly on the secondary school teaching problem. Time after time I have heard university people say that they had never before had as ready an access to high school people as the PSSC summer or in-service institute gave them. I think the reason for this is pretty clear. A teacher who is working with a college or university instructor on the materials that he is going to use in his classes is likely to be much more highly motivated than one who is taking another course."

The evidence indicates that the procedure used by SMSG in its Experimental Centers has worked quite well and suggests that an in-service training program, taught by a subject matter specialist, either before or during the first years use of the SMSG text does provide the additional mathematical preparation which the teachers require and allows the teachers to teach the course successfully and without undue effort. Furthermore the evidence indicates that this help is needed only once, and that a teacher who has been through an SMSG text once can thereafter handle the course successfully without further subject matter assistance.

In order to obtain more detailed information on the procedures used by the Center consultants, each consultant was asked to complete the following questionnaire:

Consultant Questionnaire

1. What procedures did you employ? (Check one or more)

   —— Lecture on more advanced background material.

   —— Lecture from material found directly in SMSG texts and commentary.

   —— Answer question asked by teachers.

   —— Lead group discussion seminar style.

   —— Classroom visitation and observation by consultant.

   —— Demonstration class teaching by consultant.

2. List and briefly describe other approaches you used.

3. Brief comment on effectiveness of techniques listed above.

4. Brief suggestions for more efficient use of a consultant.

5. How often did your group meet?

6. How long were the sessions?

7. Do you have any suggestions on possibly the most effective frequency of meetings and duration of sessions?

8. List any additional comments on attached sheet.

Analysis of Questionnaire Returns

It was not unexpected that there was a wide range of patterns of techniques of consultative activities. No small reason for this wide variation was variation in the mathematical background of the teachers. All of the consultants at one time or another simply answered teachers' questions about material in the book, although only one consultant listed this as his sole activity. Over half the consultants listed lecturing on advanced material, lecturing on basis of student text material, leading group discussions, and answering teachers' questions as the most effective techniques. One-third of the consultants used a combination of all four of these techniques. Some consultants reported that they had visited the teacher's classes and had used their observations as material for motivating discussions, and that this had been extremely effective. However, such activities were very time consuming.

Some of the mathematicians covered the material topic by topic while others concentrated on a few topics. It appears that sentiment favored high lighting in detail a few points which the consultant thought most important. This could be coupled with advice to the teacher on material which could be "soft-pedalled" or de-emphasized. Covering a few topics thoroughly gave the teacher a confident feeling of mastery and also served as an excellent motivation for going on to more advanced, valuable material.

Many consultants were able to motivate seminar-type discussions among the teachers on the basis of questions asked by the teachers. These questions often arose from classroom experience by the teachers. Such discussions needed guidance from the consultant to prevent them from degenerating into "bull-sessions" on classroom problems of teaching, discipline, etc. However, this did not prove to be a difficult matter, and the discussions were concerned for the most part with mathematics rather than pedagogy.

There was an even distribution between sessions which met weekly and biweekly. In many of the schools the sessions met every week for the first semester and tapered off to every other week for the second semester. Most of the sessions were two hours long with a few only one hour long and in one case they ranged up to five hours in duration.

It is interesting to note that where consultants have worked for two years with the same group we notice an increasing development of more advanced material. This is coupled with evidence from the SMSG Centers that when a teacher teaches an SMSG course for the second time the amount of in-service assistance needed and the extra time needed for preparation are both drastically reduced and in many cases disappear entirely. This has also been the experience of PSSC with its physics course.

In several Centers the consultants reacted against working with a group of teachers using new materials on several grade levels. The in-service sessions seemed to work best by having a consultant work with a group preparing to teach a particular grade level. If several grade levels of mathematics are involved the consultant would do well to meet separately with the interested groups.

Several consultants argued forceably for a reduction in the teacher's load while she is preparing to teach some of the new materials. The teacher is confronted with many hours of study and consulting session time in using the newer materials and success can possibly be destroyed by a load which detracts from these efforts.

The following remarks, taken from a report from one of the consultants, may prove interesting:

"I followed the plan of lecturing on the various units in advance of the time they were taught. In general I presented the topic in question more rigorously and went somewhat beyond the text's treatment. The lecture was designed to supplement the text. My hope was to present background and mathematical motivation. Later, as the material was being taught, there were usually questions which could be used as the basis for further discussion. The members of the group sometimes gave me questions before a meeting, and these suggested a general discussion.

"I think I may have relied too heavily on lecturing. However, I found that until the group was well into the material it was difficult to elicit discussion. One advantage of a lecture, however, is that certain difficulties can sometimes be anticipated. It avoids the need to correct explicitly a wrong idea that has been expressed. Moreover, even teachers with considerable training sometimes seem to lack, through no fault of their own, I imagine, a real understanding of the methods of mathematics. I think the consultant can and should supply, for many topics, some idea of what a rigorous, systematic approach would involve.

"I began with an idea that my main function should be to interpret new and unconventional material and supply background for it. I think that this was a mistake. For some standard topics there are two dangers. Some teachers go overboard for new ideas at the expense of necessary and important conventional skills. Others see no reason to change a tried and true method, and they revert back to it at the first sign of difficulty."

OTHER SOURCES OF INFORMATION ON IN-SERVICE TRAINING

"In-Service Education of Teachers of Secondary School Mathematics" was the theme of a conference held at the U.S. Office of Education, March 17–19, 1960. The conference had the joint sponsorship of the Office of Education and the National Council of Teachers of Mathematics.

The conferees reported many ways in which the administrators of secondary schools are helping teachers to upgrade their professional competence. A comprehensive report of the conference proceedings will be available from the U.S. Office of Education.

During the fall and early winter of 1960 the National Council of Teachers of Mathematics is sponsoring a series of regional orientation conferences in mathematics, with financial assistance from the National Science Foundation. These conferences are designed for administrators. At each of these conferences there will be a discussion of the problems of implementing new mathematics programs, including in particular the need to provide teachers with additional training in mathematics as well as various methods for doing this. A report of these conferences will be prepared and will be available from the National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

IN-SERVICE PUBLICATIONS

In order to assist those in-service teachers who wish to learn the new mathematics, and in particular the new points of view towards mathematics, which the new curriculum materials make advisable, SMSG is preparing two sets of publications which are especially designed for use in in-service programs such as those described above as well as in NSF summer institutes.

The first of these is a series of study guides for various areas of mathematics. Each of these study guides lists specific topics in the area with which it is concerned and then, for each of these topics, lists those expositions, in the currently available literature, which are simple, direct, and conveniently organized. One such study guide, concerned with the algebraic background which a teacher of a first course in algebra should possess, is already available. Manuscripts of similar study guides in geometry, logic, number theory, and probability, are now being prepared. When they are available, this information will be announced in another issue of this Newsletter.

The second series of publications consists of brief expositions of various parts of mathematics designed explicitly for in-service teachers. Five of these are already available and are described below. Additional volumes in this series are in preparation.

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TO ORDER, USE THE FORM ON THE INSIDE BACK COVER AND SEND TO School Mathematics Study Group
Box 2502 A, Yale Station
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The purpose of this Study Guide is to provide assistance to teachers who wish to improve their professional competence in modern algebra by study either individually or in small groups. It contains a list of basic topics and ideas which should be part of the mathematical equipment of teachers of introductory (9th grade) algebra. There is a short bibliography and, for each of the basic topics and ideas, detailed references to this bibliography.

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A study, written for high school teachers, of the approach to Euclidean Geometry used in the SMSG tenth-grade textbook.

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by VINCENT H. HAAG

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by B. V. KUTUZOV

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by RICHARD D. ANDERSON

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SCHOOL MATHEMATICS STUDY GROUP

Newsletter No. 10

November 1961

Reports on Student Achievement in SMSG Courses

CONTENTS

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INTRODUCTION

Any suggested curriculum revision inevitably and properly raises questions as to what losses or gains may result from change. In the case of the mathematics courses prepared by the SMSG, exemplifying greater emphasis on mathematical content and understanding, it was conceivable that reduction of time spent on drill might detract from the acquisition of manipulative skills or result in lower scores on conventional achievement tests. The legitimate concern over this expressed by educators, parents, and students has been shared from the outset by all those involved in the work of the SMSG.

A variety of studies has been undertaken. Two of these were carried out by the Educational Testing Service and the Minnesota National Laboratory.

A few comments on these studies are pertinent. First, and most important, these studies are evaluations of student achievement in SMSG courses, *not* evaluations of the SMSG courses themselves. The difference is profound. Evaluation of a course involves, among other things, value judgments by whatever authorities are appropriate concerning the short and long-range objectives of the course along with the study of the effectiveness of the course in achieving these objectives. However, in the sense used here, "evaluation" of student achievement in a course means the analysis of student scores on specified standardized tests at the end of the course.

The two reports which are reproduced here are concerned with the measurement of mathematical skills by recognized standardized tests. Both studies show that students in SMSG classes do about as well as students in conventional classes on these tests. There are small variations accompanying differences in grade level and student aptitude.

The study conducted by Educational Testing Service produced the important, albeit expected, finding that students in SMSG classes learned substantial amounts of mathematics not included in conventional courses.

It should be further noted that these studies were concerned with measurable gains in student performance over a one-year period only.

A curious and as yet unexplained discrepancy between the results of these two studies is indicative of difficulties encountered in appraisals of this sort. In the ETS study, students taught by teachers who had had previous SMSG experience did better than those taught by teachers without such experience. In the study carried out by the Minnesota National Laboratory the reverse seems to be the case.

It is to be hoped that light will be thrown on this point and that answers to other important questions will be found in a long-term study for which SMSG is now making preparations. In this study students at various grade levels will be followed for a five-year period, and changes in mathematics performance, both in and outside the mathematics classroom, will be measured. The study will include students in conventional courses as well as in SMSG and similar courses.

The results of this long-term study should provide to those interested much of the information they will need for a proper evaluation of the SMSG text materials themselves. More important, the study will provide a wealth of information on which future curriculum improvements can be based.

EDUCATIONAL TESTING SERVICE

Summary Report of the School Mathematics Study Group Curriculum Evaluation

During the summer of 1960 arrangements were made with approximately 75 schools in the United States to undertake a major evaluation of new instructional materials in mathematics. The evaluation study, designed for replication over five grade levels—grades 7, 9, 10, 11, and 12—had as its fundamental purpose the comparison of student achievement in SMSG courses with student achievement in non-SMSG mathematics courses.

Evidence pertinent to three crucial questions was secured. The three questions follow:

1. Does the SMSG curriculum detract from student achievement with respect to traditional "mathematical skills"?

2. Does the SMSG curriculum result in a measurable extension of developed mathematical ability beyond that of conventional mathematics instruction?

3. How effectively is the SMSG curriculum communicated to students at various levels of scholastic ability?

Evidence pertinent to question *one* was secured in the following manner. A group of teachers (CA), selected at random from a group of teachers willing to teach the SMSG curriculum for the first time, provided their students with conventional mathematics instruction. A second group of teachers (EA), selected at random from a group of teachers willing to teach the SMSG curriculum for the first time, provided their students with mathematics instruction based on SMSG materials. There were approximately 30 teachers in each of the two groups, CA and EA, at each of five grades, 7, 9, 10, 11, and 12.

Students of CA teachers and students of EA teachers were administered common tests of scholastic aptitude and knowledge of mathematics in the fall of 1960, and common tests of traditional mathematics and SMSG mathematics in the spring of 1961. The tests were designed for the various grade levels and curricula involved in the study.

Before comparing the mathematics achievement of SMSG and non-SMSG students we shall distinguish between the statistical significance and the practical significance of educational results. A statistically significant difference is customarily reported when the probability that such a difference can be attributed to chance or random factors is less than 1 in 20 or 1 in 100. Although the question of practical significance is seldom dealt with in curriculum studies, it is legitimate to say that a practically significant difference occurs when gains or losses in knowledge or skills are demonstrable.

Relative to question *one* we are concerned with the comparative achievement of the two groups of students described above on tests of traditional mathematical skills. At the seventh-grade level the students of CA teachers have an average achievement score of 37.3. This is 2.6 raw score points *higher* than the average achievement score for 34.7 for students of EA teachers. The two average achievement scores have been adjusted for initial differences among students in scholastic aptitude and knowledge of mathematics. The 2.6 difference is statistically significant, but its practical significance is dubious. The difference is about one-sixth of the standard deviations and represents fewer than three additional test items answered correctly.

At the ninth-grade level the students of CA teachers have an average achievement score of 20.08. This is 7.92 raw score points *higher* than the average achievement score of 12.16 for students of EA teachers. The two average achievement scores were *not* adjusted for initial differences in aptitude and mathematics achievement, since it was not statistically feasible to make these adjustments.[1] Although the students of CA teachers are 7.92 points higher on the average than the students of EA teachers, the CA group is also considerably *higher* on both scholastic aptitude and prior knowledge of algebra than the EA group. Since these two variables are highly correlated with mathematics achievement, it can be seen that the observed difference of 7.92 could easily have been reduced to a difference of 2 or 3 raw score points if the scores were adjusted. If this line of reasoning is correct, one may again question the practical significance of the obtained differences.

At the tenth-grade level the students of CA teachers have an average achievement score of 20.27. This is 6.25 raw score points *higher* than the average achievement score of 14.02 for students of EA teachers. The two average achievement scores were *not* adjusted for initial differences in scholastic aptitude and mathematics achievement. Again it was not statistically feasible to do so. Herein, the CA group was slightly lower in academic aptitude and somewhat higher in initial knowledge of geometry. Hence, it is highly likely that the observed difference of 6.25 would have been only slightly altered if the scores had been adjusted. In this case there is little doubt that the advantage of the CA group over the EA group is both practically and statistically significant.

At the eleventh-grade level the students of CA teachers have an average achievement score of 21.8. This is .7 of a raw score point higher than the average achievement score of 21.1 for the students of EA teachers. The two average achievement scores were adjusted for initial differences in aptitude and achievement. Herein, the achievement difference between the two groups is neither practically nor statistically significant.

Twelfth-grade students of CA teachers have an average achievement score of 19.4. This is 1.4 raw score points *less* than the average achievement score of 20.8 for students of EA teachers. The two average achievement scores were adjusted for initial differences in aptitude and achievement. The difference between groups is not statistically significant and we have no evidence that suggests a practically significant difference.

On the basis of the data described above it can be seen that, over-all, students exposed to conventional mathematics have neither a pronounced nor a consistent advantage over students exposed to SMSG mathematics with respect to the learning of traditional mathematical skills.

[1] The analysis of covariance technique, or the adjustment on achievement for initial differences in relevant variables, can only be employed if two conditions are met.

a. There should be no statistically significant difference between the standard errors of estimate for the two groups. If there is, then an uncontrolled source of variation is present.

b. There should be no statistically significant difference between the slopes of the two regression lines. If there is, then it would be illogical to adjust scores all along the slopes by an equal amount.

In most cases there were statistically significant differences between the standard errors of estimate, or the slopes, or both, for the two groups being compared.

The same conclusion is reached when we engage in a second set of comparisons. The achievement of students of CA teachers, described above, is compared with the achievement of students of EB teachers. A group of teachers (EB), selected at random from a group of teachers who had previously taught SMSG mathematics, provided their students with SMSG mathematics instruction.

At each of the grade levels 7, 10, 11, and 12 the students of EB teachers have higher average achievement scores in traditional mathematics than the students of CA teachers. However, the students of EB teachers also have higher average scores in initial aptitude and mathematics achievement than the students of CA teachers at all grade levels. Hence, it can be concluded that if the achievement scores were adjusted for the two groups, the advantage of the EB students would be lessened in grades 10, 11, and 12. However, the advantage of the EB students at grade 7 is too large to be significantly reduced by adjusting scores. Conversely, the CA students at grade 9 would retain their advantage over EB students even if the scores were adjusted. That is, in spite of the fact that EB students at grade 9 have somewhat higher scores on initial aptitude and achievement than CA students at grade 9, the CA students still manage to achieve a higher score on traditional mathematics. This datum parallels that found at the tenth-grade level in the earlier comparison, the comparison between EA and CA students. The datum ought to be regarded as both practically and statistically significant. In the light of the above comparisons we again conclude that, in general, students exposed to conventional mathematics have neither a pronounced nor a consistent advantage over students exposed to SMSG mathematics with respect to the learning of traditional mathematical skills.

Additional evidence pertinent to question one was secured by observing the conventional mathematics test scores of three groups of students—the EA and EB students, described earlier, and an historical group of students.[2] Comparisons among these three groups were restricted to three grade levels, 9, 10, and 12, since these were the only areas in which data were available for the historical group.

At the ninth-grade level the traditional mathematics achievement scores of EB students were highest (18.7), the scores of historical students were next highest (16.5), and the scores of EA students were lowest (12.2). For the tenth grade the scores of EB students were highest (21.6); the scores of historical students were next highest (19.3), and the scores of EA students were lowest (14.0). The positions are altered somewhat at the twelfth grade. Here the scores of the historical group were highest (27.6), the scores of the EB group were next highest (23.0), and the scores of the EA group were lowest (21.1).

A striking feature of the data derived for these three groups is the relationship between aptitude and achievement test scores. That is, in *no* case is the relative position of a group, as determined by rank ordering on achievement, reversed when we observe the relative positions of the groups as determined by rank ordering on aptitude. At each of the three grade levels the highest group in aptitude is the highest group in achievement; the medium group in aptitude is the medium group in achievement; and the lowest group in aptitude is the lowest group in achievement.

We may reasonably conclude that if the achievement test scores were adjusted to take into account initial differences in aptitude, then the observed differences in achievement test scores would be diminished.[3] The probability is very high that there are neither statistically significant nor practically significant differences in the scores of these groups that could be attributed to differences in curricula. Hence, conclusions relative to question *one* based on comparisons between contemporary SMSG students and historical groups taught conventional mathematics are quite consistent with conclusions derived from the comparisons described earlier.

Evidence relative to question *two* was secured in the following manner. The students of CA teachers were compared with students of EA teachers on the basis of their performance on SMSG tests. The students of EA teachers had higher average achievement scores at *all*

[2] Students taught conventional mathematics at the grade level indicated during the academic year 1950–51. Their scores on conventional mathematics achievement were obtained from records made available by the Educational Records Bureau and the Educational Testing Service.

[3] Adjustments in achievement test scores due to differences in aptitude were not made, for reasons noted earlier in this report.

grades. Even if these average achievement scores had been adjusted for initial differences in aptitude and mathematics achievement, such adjustments were not feasible; the students of EA teachers still would have retained significant advantages in SMSG achievement. The advantage becomes even more significant in favor of SMSG students when EB and CA students are compared. Thus, we conclude that students exposed to SMSG instruction acquire pronounced and consistent extensions of developed mathematical ability beyond that developed by students exposed to conventional mathematics instruction.

Data relevant to question *three* were derived by plotting SMSG test score distributions according to differing SCAT levels. Then, overlap among SMSG test scores was sought for students of high, medium, and low scholastic aptitude. From the charts it can be seen that overlap among SMSG scores is modest and normal for grades 7 and 9. The overlap increases at grade 10, and becomes pronounced at grades 11 and 12. Charts illustrating conventional mathematics test score distributions according to differing SCAT levels indicate a comparable phenomenon for grades 7 and 9, but not for grades 10, 11, and 12.

Several hypotheses are suggested by these data. First and foremost is the fact that scholastic aptitude is far from the whole story in predicting achievement in SMSG. The necessity for additional predictors of SMSG achievement becomes particularly acute in the upper grades. Additionally, the large range of achievement scores for all SCAT levels at all grade levels casts doubt on traditional means of selecting students for ability grouping in mathematics instruction. Finally, there is positive evidence to suggest that students at all SCAT levels can learn considerable segments of SMSG materials.

An additional facet of this study was the concern for results that might be attributable to Hawthorne or experimental effect. In order to establish some control over the influence that participating in an experiment is alleged to exert on experimental results, an additional comparison of traditional mathematics achievement was made.

A group of teachers (CC) was randomly selected from the populations of mathematics teachers from the large school systems who participated in the study. Their participation included only the administration of SCAT and achievement tests of conventional and SMSG mathematics

in the spring of 1961. These teachers did not know that they would be asked to participate in the study until shortly before the spring administration of tests. Hence, it is hypothesized that their instruction was not influenced by knowing that they were in an experiment.

Comparisons of achievement on conventional and SMSG tests for students of CA and CC teachers indicate unequivocally that there is no advantage in favor of students of CA teachers, those teachers who knew they were in an experiment. The CC teachers provided instruction in conventional mathematics only.

Roland F. Payette
Assistant Director
Curriculum Studies

MINNESOTA NATIONAL LABORATORY
Evaluation of SMSG, Grades 7–12

I. INTRODUCTION

For a description of the Minnesota National Laboratory, see SMSG Newsletter No. 2.

This is a report on experimentation carried on by the Minnesota National Laboratory with the SMSG materials for grades 7–12.

The real work began in the academic year 1959–60. We conducted a controlled experiment in grade 7, dry runs in grades 8–12, and a pilot study of the use of the SMSG 7th-grade course in grade 6. We also conducted a careful experiment on the effects of grouping in grade 7.

In 1960–61 we continued our experimentation as part of a joint project to investigate teachers' characteristics in relation to students' learning. We also conducted an experiment on alternative treatments of drill in grades 7, 9, and 11, and a preliminary evaluation of the SMSG materials for the non-college-bound in grades 7 and 9.

Application forms for participation by teacher and by school were sent to every superintendent and to headmasters of private schools in Minnesota. Since participation by both teacher and school was entirely voluntary and by application, the teacher and school populations were undoubtedly biased at least with respect to their attitudes toward experimentation.

A crude measure of teacher qualifications was set up in terms of experience, grades in undergraduate and graduate courses in mathematics, activities in professional organizations, and contributions to the advancement of mathematics teaching. The population of teachers was stratified according to this measure of qualifications, and schools were stratified by population of community. A random selection was made from each stratum. Thus, we had both well and poorly qualified teachers, and schools from large cities, small towns, and rural areas.

In 1959–60 mathematicians from the colleges and junior colleges in the state held two regional meetings per month—one for teachers in grades 7–9, and one for teachers in grades 10–12. The college teachers were instructed to answer specific questions and to provide a forum for discussion, but not to give lectures or otherwise to provide in-service training as at the SMSG centers in other parts of the country.

Of course, some of the teachers had had in-service education at summer institutes or at summer workshops conducted in Minnesota in 1959.

Each September we gave pre-tests for aptitude, using DAT for grades 9–12 and SCAT for grades 7–8, and achievement, using STEP in all grades. We tested again for achievement the following May and the following September. In 1959–60 we also gave attitude tests in grades 7 and 10. The report on the interpretation of these results will be made by another group.

In the study on teacher characteristics started in 1960–61, Flanders has been studying the interaction between teachers and classes, and Torrance has been analyzing the teachers' reports from the point of view of their critical judgment, creativity, and other factors.

II. SUMMARY OF RESULTS

Generally speaking, students in the SMSG classes did at least as well as would be expected in achievement. For example, in 1959–60 in grade 11 on the pre-test about 5% of the students were below the median according to national norms for 11th-graders, but a year later less than 1% scored below the median for 12th-graders. Thus, about 80% of the students who were initially below the national median were above the national median the following September.

In grades 6 and 9 the SMSG courses (remember that in grade 6 we used the SMSG 7th-grade course) gave especially good results for the students in the bottom quartile. Thus, in 9th grade in 1959 we started with about 16.5% in the bottom quartile according to national norms for 9th-graders. The following September less than 10% were in the bottom quartile for 10th-graders, and, in fact, less than 17% were in the bottom quartile according to national norms for 11th-graders!

In all grades the high-ability students also did well. For example, we started in 1959 with 68.6% of the 10th-graders in the top quartile according to national norms, and a year later 66.9% of these students scored in the top quartile according to 12th-grade norms.

In general, the pattern of achievement the following September compared well with students around the country at the next higher grade level.

In the 7th grade we conducted an experiment with 13 teachers each teaching one SMSG class and one conventional class. In each school neither or both classes were grouped for ability, and we made a random choice, after the sectioning was done, of which was to be the experimental class. On retesting the following May there was a significant difference in favor of the SMSG classes. On retention testing the following September we found a slight difference in favor of the SMSG classes, which was not statistically significant.

In one school the counselor, J. Mikkelson, conducted in 7th grade a careful experiment on the effects of grouping, replicating an experiment done the previous year using conventional materials. Differences in grouping had no significant effect on the achievement of high-ability students, but low-ability students did significantly better in sections with no high-ability students than in heterogeneously grouped classes.

We found few significant correlations between student achievement and the factors of teacher qualification which we measured. We have some indication that the teacher's grades in undergraduate and graduate mathematics courses are significant for his effectiveness with the SMSG 12th-grade course. In the other grades the kind of information which one can get from a transcript or an application form seems to be irrelevant to the teacher's effectiveness with SMSG.

The preliminary results of the Flanders–Torrance–Rosenbloom study on teacher characteristics do give some clues to the factors in teacher effectiveness. When asked such questions as "What criticisms do you have of this section?" "What are your suggestions for improvement?" "What difficulties did you run into?" "How did you overcome them?" the effective teachers were, in general, the ones who gave specific answers, while the poor teachers were the ones who answered in vague generalities, such as "I like it," or "I don't like it," or "It is easy for bright kids but too hard for dull ones." The effective teachers had more flexible behavior patterns in the classroom, talked less in proportion to the time the students talked, less frequently told students what to do and more frequently got the students to suggest how to attack a problem, and more frequently elicited ideas from the class. It seems unimportant whether the teacher likes SMSG or not, but it does seem important that he be open-minded and interested in trying something new. Creativity, critical judgment, and flexibility are the most important factors in teacher effectiveness which we have found so far.

In some cases a teacher who was very effective the first year apparently became sold on the new course and was quite uncritical the second year. When this happened, student achievement declined correspondingly.

We have scattered reports—which have not yet been checked systematically—that where the decision to use SMSG outside of our experimentation program was made by the administrators and imposed on the teachers, the results were unfavorable.

III. DATA AND ANALYSES

A full technical report on our 144 pages of analysis of data will be published elsewhere. Here we shall try to give a sample to illustrate the kinds of information which we have obtained.

A. Grade 6

In 1959–1961 we had several teachers using the SMSG 7th-grade course in 6th grade. While five of the seven classes in 1959–60 and all three of the 1960–61 classes were described by their principals as "average," our test data indicate that they were definitely of better than average ability. Two of the teachers had had special mathematical training; these were not the ones with the two high-ability classes in 1959–60. The others were good ordinary elementary teachers with the usual minimal mathematical background. The classes covered about eight chapters of the text.

In the following table we list the proportions of the 65 students for whom we have complete test data in 1959–60 who scored below the indicated percentiles, according to national norms, on the indicated tests. For the post-test and the retention test we compared with the national norms for fall administration to 7th-graders ("SCAT-T" means "SCAT-total"):

	Pre-tests, Sept. 1959		Post-test, May 1960	Retention, Sept. 1960
Percentile	SCAT-T	STEP	STEP	STEP
25	16.9	24.6	7.7	8.2
50	33.8	44.6	27.7	21.5
75	64.6	65.6	55.4	43.1

We have test data on 61 students in the 1960–61 classes:

	Pre-tests, Sept. 1960		Post-test, May 1961
Percentile	SCAT-T	STEP	STEP
25	1.9	13.0	10.7
50	13.1	25.4	16.0
75	43.4	43.5	37.7

The teachers without special training in mathematics did as well as the two with special training.

In comparison with national norms for 8th-graders (fall administration) we have the following proportions for the indicated percentiles:

	1959–60	1960–61
Percentile	Retention, Sept. 1960	Post-test, May 1961
25	15.4	13.2
50	33.8	32.8
75	58.5	51.1

Thus, the students compared well with 8th-graders nationally at the end of the course.

For 1960–61 we obtained the following regression equation as the best linear prediction of post-test score (x_5) on pre-test scores ($x_1 =$ SCAT − Verbal, $x_2 =$ SCAT—Quantitative, $x_4 =$ STEP):

$$x_5 = -31.4 + .269\, x_1 + .348\, x_2 + .516\, x_4$$

The rather small coefficient of x_1 suggests that the children's reading ability was not as great a factor in their achievement with the SMSG course as some educators think.

The discrepancy between pre-test scores on aptitude and achievement is consistent with the frequent finding by educators that children's achievement in mathematics in our schools usually lags behind their aptitude.

Note the marked improvement of the pupils in the bottom two quartiles.

B. Grade 7

In 1959–60 we had 13 schools with one experimental and one control class taught by the same teacher. The

class assignments were made in the spring of 1959, while the random choice of which of the two sections was to be the experimental was made in September 1959. In only two schools could we arrange for random assignment of pupils to the two sections. Thus, an analysis on the basis of the class as the experimental unit is completely justified, while an analysis on the basis of the pupil as the experimental unit is probably, but not certainly, justified.

On the post-tests of May 1960 we found significant differences, at the 1% level, in favor of SMSG both in achievement adjusted for pre-test achievement score and in gains in achievement adjusted for pre-test on aptitude. Thus, there is less than one chance in one hundred that differences as large as those which we found could be attributed to chance.

For 11 of the 13 teachers the differences were also in favor of SMSG, and for 7 of them the differences were significant at the 5% level. In neither of the two cases where the differences were in favor of the control class were they significant.

For 6 of the 7 teachers who had previous SMSG experience and for 5 of the 6 without SMSG experience, the differences were in favor of SMSG. For 4 of the 7 teachers in the first group and for 3 of the 6 teachers in the other, the differences were significant.

We found a negligible interaction between teacher and treatment. In other words, the differences between achievement of experimental and control classes did not vary significantly from teacher to teacher, in spite of the enormous variation in the general effectiveness of teachers.

On the retention tests in September 1960 there were still differences in favor of SMSG, but they were no longer significant.

If we break down the student population into low, middle, and high thirds on the basis of the pre-test scores on SCAT, we obtain the following figures for the means on the post- and retention tests with STEP, as adjusted for the pre-test scores:

	Post-test, May 1960		Retention test, Sept. 1960	
	Experimental	Control	Experimental	Control
High	276.01	272.54	277.62	274.36
Middle	270.53	268.75	272.41	272.43
Low	266.45	264.55	268.72	266.66

The differences for all ability levels are consistently in favor of the SMSG classes, but they are not significant at the 5% level.

The regression lines of retention test scores (y) versus pre-test achievement scores (x) were:

$$y = 39.055 + .8892\ x\ \text{(SMSG)}$$
$$y = 21.069 + .9536\ x\ \text{(Conventional)}$$

These best linear prediction formulas for achievement at the beginning of grade 8 on the basis of achievement at the beginning of grade 7 give a higher prediction for SMSG than for the conventional course for students whose pre-test score is ≤ 279.3, and beyond that favor the conventional course. Thus, we could expect a better performance in the SMSG course for all but about the top 1% of the pupils. The confidence bands are, however, so wide that this finding should be interpreted with caution!

In 1960–61 we have analyzed data on 312 pupils in the SMSG 7th-grade classes, and found the following proportions at or below the indicated percentiles on the indicated tests:

	Pre-tests, Sept. 1960		Post-test, May 1961
Percentile	SCAT-T	STEP	STEP
25	1.9	6.8	4.4
50	21.2	15.0	17.0
75	46.2	45.8	41.5

The bottom quartile did slightly better than expected, as did the students in the 50–75th percentile range.

In one junior high school the counselor, Mr. J. Mikkelson, conducted a two-year study on the effects of grouping. The top 20% (70 pupils) of the 7th-graders were randomly divided into an experimental and a control group. Half were assigned to one class, while the others were distributed randomly to half of the other 7th-grade classes. The other 80% were randomly distributed into classes with and without high-ability pupils during each period of the day. Thus, time of day was also controlled. In 1958–59 this experiment was conducted with the conventional curriculum, and in 1959–60 with the SMSG course. In 1959–60 a small class, using conventional materials, was reserved for transfer students. During both years teachers were randomly assigned to different types of classes so as to control teacher variability.

On testing with STEP, and with the California Arithmetic Reasoning and Fundamentals tests, we found no significant difference in achievement between the experimental and the control groups of high-ability pupils. These pupils gained, on the average, between .5 and .7 grade equivalents more on the California tests than the similar group did in the conventional course the year before. It is not certain that the groups for the two different years are strictly comparable. On most of these measures there was no significant difference in achievement of the other 80%, the pupils between the classes which had some high-ability students and those which did not. Only in gains on the California Reasoning test was there a significant difference at the 5% level, and this was in favor of the classes without high-ability pupils. But this difference was still quite small in comparison to the large differences ascribable to the interaction of teacher with the treatment. In other words, some teachers are much more effective with one type of class than with another, and this is more important than any of the other factors we investigated.

C. Grade 8

In the following tables we list the proportions of the 335 students for whom we have complete test data in 1959–60 who scored below the indicated percentiles, according to national norms, on the indicated tests. For the post-test and the retention test we compared with the national norms for fall administration to 9th-graders:

	Pre-tests, Sept. 1959		Post-test, May 1960	Retention, Sept. 1960
Percentile	SCAT-T	STEP	STEP	STEP
25	3.0	3.6	2.7	3.9
50	18.9	11.1	12.3	13.2
75	42.0	28.8	40.2	33.3

We have analyzed the following test data on 479 students in 1960–61:

		Pre-test, Sept. 1960	Post-test, May 1961
Percentile	SCAT-T	STEP	STEP
25	6.3	9.3	4.9
50	16.1	17.3	13.6
75	40.3	35.8	34.1

Thus, the students did about as well as might be expected, except possibly for the highest quartile in 1959–60.

In both years the materials arrived late for the beginning of the school year, so that the comparison may not be entirely fair. Also, the 8th grade is the one SMSG sample text which assumes the SMSG course for the preceding year, so that a proper comparison should be with the grade 7–8 sequence.

We shall present the data for grades 9–12 in the above format.

D. *Grade 9*

1959–60—complete data on 109 students ("DAT-T" means "DAT—Verbal + quantitative"):

	Pre-tests, Sept. 1959		Post-test, May 1960	Retention, Sept. 1960
Percentile	DAT-T	STEP	STEP	STEP
25	11.0	16.5	8.9	9.9
50	27.5	30.3	33.0	31.2
75	60.5	51.4	58.7	53.2

1960–61—data on 262 students:

	Pretests, Sept. 1960		Post-test, May 1961
Percentile	DAT-T	STEP	STEP
25	6.0	5.7	9.5
50	22.5	14.8	22.9
75	40.1	33.4	50.9

The results are inconclusive.

In 1959–60 there was a marked improvement for students in the bottom quartile, but students in the top quartile did slightly worse than expected. In 1960–61 the students did worse than would be expected.

E. *Grade 10*

1959–60—complete data on 433 students:

	Pre-tests		Post-test	Retention
Percentile	DAT-T	STEP	STEP	STEP
25	1.2	3.1	3.2	2.9
50	12.2	9.9	11.3	7.2
75	34.2	31.4	34.4	29.1

1960–61—data on 472 students:

	Pre-tests		Post-test
Percentile	DAT-T	STEP	STEP
25	3.0	2.1	1.8
50	9.3	7.9	7.5
75	23.5	26.8	28.0

Again the students did about as well as might be expected. The highest-ability students (top quartile) did slightly worse than expected in the post-tests, but on retention the following fall did slightly better than expected. Although the number in the bottom quartile (22 in the two years) may have been too small for significant conclusions, the results for the 80 students who were below the national median indicate that low-ability students do slightly better in the SMSG course than would be expected.

Our regressions of post-test and retention scores on pre-test data do not indicate that verbal ability is a particularly important factor in the students' achievement in the SMSG 10th-grade course.

It is interesting to compare the retention test data with the national norms for fall administration to 12th-graders:

Percentile	Proportion Sept. 1960
25	3.2
50	11.3
75	33.9

Thus, the students generally compared well with 12th-graders according to the publisher's data.

F. *Grade 11*

1959–60—complete data on 228 students:

	Pre-tests		Post-test	Retention
Percentile	DAT-T	STEP	STEP	STEP
25	2.2	0	.4	.9
50	4.4	5.3	1.8	.9
75	29.4	22.4	12.3	7.9

1960–61—complete data on 416 students:

	Pre-tests		Post-test
Percentile	DAT-T	STEP	STEP
25	1.4	1.6	2.1
50	8.2	6.4	6.7
75	28.1	22.4	21.3

We see that the students did about as well as might be expected, and that the results in 1959–60 were much more favorable to SMSG than the results the following year. This was in spite of the fact that none of the teachers had previous SMSG experience the first year, whereas 15 of the 17 had previous SMSG experience the second year. Altogether we had about 35 students below the national median, and they seem to have done as well as would be expected.

Again there was no evidence that verbal ability was a particularly important factor in student achievement.

G. Grade 12

1959–60—complete data on 370 students:

	Pre-tests	Post-test
Percentile	STEP	STEP
25	0	0
50	3.4	.8
75	10.2	6.0

1960–61—complete data on 328 students:

	Pre-tests		Post-test
Percentile	DAT-T	STEP	STEP
25	0	0	0
50	0.6	1.5	1.1
75	16.5	10.2	6.9

We have inadequate data on the performance of these students on college entrance tests.

The students did slightly better than might be expected. There were too few in our sample who were initially below the national median to draw any conclusions about the performance of low ability students.

It should be noted that the SMSG 12th-grade course was supplemented by the material on trigonometry from the SMSG 11th-grade course. Thus, there has not yet been a test of the SMSG 12th-grade course with a full year allotted to this sample text.

H. Teacher Characteristics

Only in 12th grade was there any evidence that student achievement might be significantly correlated with any of the factors of teacher qualification which we could obtain from an application form or a transcript, such as experience, grades, activities in professional organizations, and contributions to the advancement of mathematical education. There was some indication, far from conclusive, that the teacher's grades in undergraduate and graduate mathematics courses might be a factor in his effectiveness with the SMSG 12th-grade course. Otherwise our analyses gave no significant results.

IV. INTERPRETATIONS AND CONCLUSIONS

The STEP tests were chosen to measure achievement since they have a higher proportion of nonroutine problems than any of the other standard achievement tests. Nevertheless they are adapted to the conventional curriculum, and do test the skills and concepts emphasized in that curriculum. Presumably the students learned also some of the new content of the SMSG course, but in this report we are comparing the students' performance only with respect to goals which are common to the two curricula and for which they are usually tested.

Note that skill in deductive reasoning is a goal both of the conventional geometry course and of the SMSG courses in grades 9–12. Yet we know of no standard achievement test which adequately measures these skills.

In grade 7, where we conducted a controlled experiment, the students in the SMSG course did significantly better than those in the control classes on the post-test in the spring, but only slightly better on the retention test the following fall. This is good in the sense that the pupils in the experimental course learned the standard skills at least as well as the pupils in the conventional course, and probably other things as well. The result is disappointing in another sense, since one of the hypotheses of SMSG was that by stressing understanding of concepts rather than mechanical mastery of techniques, they would impart to the children a coherent intellectual structure, and that consequently the pupils would learn the skills better and retain them longer. It may be that the effect is cumulative and would be measurable only three or four years later.

These conclusions apply to all ability levels—high, middle, and low—whether in homogeneously or heterogeneously grouped classes. On the retention tests the low and the high thirds still did better in the SMSG course than in the conventional, but the middle third did about the same.

Is the SMSG 7th-grade course unsuitable for low-ability students? Our evidence is that the answer is "No." Teachers and administrators tend to forget that low-ability students do poorly in the conventional courses. If anything, our results indicate that the low-ability students profit more from the SMSG course, in comparison to the conventional, than the other pupils, and that the course does not place an undue strain on their verbal ability.

Does the SMSG 7th-grade course, together with homogeneous grouping, constitute a program for the gifted? Our evidence is that the answer is "No." In fact, we found no significant difference between the achievement of high-ability students in homogeneously grouped and in heterogeneously grouped classes, whether the SMSG or the conventional course was taught. No matter which curriculum is used, homogeneous grouping does not benefit high-ability students unless something special is done to take advantage of the grouping. Either the class should progress faster, or the course should be enriched, for example, with the SMSG supplementary materials.

Does the teacher need to be especially trained or highly qualified to teach the SMSG 7th-grade course successfully? Our evidence is that the answer is "No." The above comparisons hold both for teachers who had previous SMSG experience and those who had none, and both for highly qualified and poorly qualified teachers. The teacher's formal qualifications, as measured by experience and course grades, are not important factors in his effectiveness. What seems to matter is his attitude, his willingness to try something new, and his constant exercise of critical judgment. As a result of our experience, we would suggest the hypothesis that teaching one of the new courses is itself the best form of in-service education.

Is it feasible to teach the SMSG 7th-grade course in 6th grade? Our evidence is that the answer is "Yes." One can expect to cover about eight chapters. Children of all abilities seem to profit from the course. An ordinary elementary teacher can handle it successfully if there is a mathematical consultant available. The children compare with 8th-graders nationally at the end. We have no comparison between this treatment and the SMSG 6th-grade course.

The above conclusions apply, in general, also to the SMSG courses in grades 8–12. In these grades we had no control classes, and can only make comparisons with the publishers' data on the performance of students in the conventional classes. We did not find any ability group in the SMSG course for any grade that did much worse than would be expected according to national norms. In all cases the students did at least as well as would be expected, and in some cases much better. Only in 12th grade did we find any evidence that the teacher's formal preparation might be a factor in his effectiveness with the SMSG course, and even here the evidence is inconclusive.

Are the SMSG courses primarily for high-ability students? This seems to be an unwarranted assumption by schoolmen. In grades 7–8 we asked for students of all ability levels, and in grades 9–12 we asked for general college-capable students. According to our pre-tests, even classes which were described by the administrators as "average" consisted actually of mostly high-ability students. Of course, a self-selection normally occurs in grades 11 and 12, and may be the dominant factor in these grades.

Nevertheless, in grades 6–10 we had enough students in the bottom quartile, and in grade 11 enough below the median, according to our pre-tests, to draw significant conclusions. In every case the low-ability students did at least as well as would be expected. In grades 6 and 10, and perhaps in grade 11, they did much better than would be expected.

The high-ability students generally did as well on the retention tests as students about a grade ahead. For example, the 10th-graders, when tested the following September, compared well with 12th-graders nationally.

In general, the students did better on the retention tests in September than they did on the post-tests the previous spring. This may be some evidence that in some of the experimental courses SMSG did succeed in giving a coherent structure which facilitated retention and relearning after a brief review. Unfortunately, we cannot make a clear comparison since we have no adequate data on the performance of students in conventional courses.

We should not like to encourage any misinterpretation of our results on the relations between teacher qualifications and student achievement. We do not believe that subject-matter competence is irrelevant to a teacher's effectiveness. Our evidence merely indicates that the usual ways of measuring this competence, in terms of experience, credit hours, and course grades, are not satisfactory. In our future research we hope to find more relevant measures of a teacher's mathematical competence. In the meantime, we should certainly encourage the improvement of certification requirements and the participation of teachers in in-service programs.

Our results have a direct bearing on a quite different problem. Suppose a school administrator asks, "Can I use SMSG or similar courses in my school? My teachers are inexperienced. Or their background is poor. Few have attended summer institutes." On the basis of our present evidence we must answer, "We know of no reason why you cannot, providing your teachers have the proper attitude. If they are willing to try new courses, and if they try to judge the materials critically, there is no reason to expect trouble. If you can arrange for a mathematical consultant or an in-service program during the first year, then so much the better."

We might mention that the variability in the effectiveness of teachers from one to another, and even of the same teacher from one class to another, is very great. One must be very cautious about drawing conclusions where there are no direct comparisons, unless the teacher samples are quite large and randomly chosen, even when they are supposedly "matched."

We must also emphasize the tentative nature of our conclusions in another direction. The true payoff on the SMSG courses in grades 7 and 8 may not be observable until the 10th grade, and similarly the proper comparison for the SMSG courses in grades 9–12 may be in college or post-college achievement. Any conclusions on the basis of our research to date must be considered only as a working hypothesis to the effect that "SMSG doesn't seem to do any harm to students, and may be good for some of them."

PAUL ROSENBLOOM
Director
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SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 12

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Financial Support of the School Mathematics Study Group has been provided by the National Science Foundation

If you are not now on our mailing list but wish to receive further issues of this NEWSLETTER, please request, by means of a post card, that your name be added to the mailing list.

ANNOUNCING THAT NEWLY DEVELOPED TEXTBOOKS ARE AVAILABLE FOR 1962-63

GENERAL CONSIDERATIONS FOR USERS

The primary purpose of this issue of the SMSG Newsletter is to announce the availability, for general classroom use, of some additional texts prepared by SMSG. Therefore, some general remarks are in order to school administrators contemplating the use of any of these texts in their schools.

I. IN-SERVICE ASSISTANCE TO TEACHERS IS NECESSARY

Most school teachers were never given the opportunity in their pre-service training to see mathematics from the point of view expressed in these texts. Our experience is that without some in-service assistance, devoted to subject matter, most teachers find it no easy task to teach from such texts, and their students benefit little. But with such in-service assistance, most teachers are able to teach these new courses successfully, and their students not only do well on traditional skills but also gain a better understanding of basic concepts and principles.

An important source of the needed additional mathematics training is the program of summer, academic year, and in-service institutes sponsored by the National Science Foundation. However, there are not enough of these, particularly for elementary school teachers, to satisfy the demand. Therefore in many cases, local school systems will need to arrange their own in-service programs.

The next section of this newsletter draws upon SMSG experience in describing the elements to be considered in developing an in-service education program. For 1961-62 there are more than 110 such programs involving about 1500 teachers, cooperating in the development of SMSG texts.

II. DECISION TO USE SMSG TEXTS IS AT THE LOCAL LEVEL.

It is the function of SMSG to prepare improved texts, to test them in classrooms, and then to make them available. The decision to use these texts, and the implementation of the decision, is entirely up to the local school systems.

SMSG cannot provide any assistance to school systems using its texts. In fact, a basic principle in America is that education is locally controlled. SMSG, which receives all its financial support from the Federal Government through the National Science Foundation, wishes to do nothing which might be interpreted as an attempt to influence this local control of education.

BUILDING AN IN-SERVICE EDUCATION PROGRAM IN MATHEMATICS

Our best school systems have long recognized the value of in-service education programs. In fact, such activity has its counterpart in industry, business or any sphere of activity faced with change.

Competent observers, in a wide range of school situations, levels and locations, have a consensus that in-service education has played a strongly supportive role in the success of teachers undertaking the use of new instructional materials for school mathematics.

It is both fortunate and unfortunate that no pattern or formula can be stated that will guarantee a program that is "successful". Unfortunate, that a successful program may not be assured by following the pattern, but fortunate for it is most probable that a single pattern would require specific ingredients not available in many situations. Fortunately there is no ritual with its complement of roles needed for a successful program.

What are the guidelines in establishing a mathematics in-service education program? Many of them would apply to any education program for adults, others are clearly related to the present conditions of development in mathematics education. Both are important.

► Concerning content, it is desirable for a teacher to understand the mathematics that precedes and follows the area for which his instruction is responsible. Change, as well as limited pre-service programs, is reflected in the shortcomings of teachers' backgrounds. For elementary school teachers there is also the preparation dilemma of covering many fields. Emphasis on mathematics is rare in these multi-disciplinary programs. But if it had been, new ideas in mathematics, now important in elementary school mathematics, would not have been included.

What is the framework of existing successful in-service education programs for teachers using these new materials?

► The focus is on understanding ideas in mathematics. The elements are teachers who desire to use new instructional materials, resource persons, time to work, and materials to work with.

► Resource personnel is where you find it. College faculties are excellent places to look. High schools, too, have provided fine resource people. Some will be found on elementary school staffs and in the central educational offices of districts, counties and states. Persons who are now recognized as highly capable in helping teachers, nearly always have increased their competency by working at that job. This leads to the next point on resource personnel.

Schools have developed such resources from within their staffs. Having such personnel increases flexibility in planning, assures an ongoing effort, and is a prudent asset. NSF Institutes have greatly aided development of local resources. A resource "team" joining school staff and "outside" members achieves a working-training situation for each team member. Considering many factors, it is probable that the best available mathematician teamed with the best available mathematics teacher are the likeliest combination for resource personnel. Some schools are exceptional in having access to excellent resource personnel, others face a paucity of talent; a decision to use new materials reflects an estimate favorable to improvement in a specific setting.

► The teachers engaged must share in the belief that the in-service work is a part of an effort which will increase the effectiveness of classroom instruction. The week-by-week movement of the project must sustain this belief. While selecting volunteers is a simple way of starting with such a group, it is not the only successful way. Good teachers may rightfully require a demonstration that new materials do improve instruction; on this ground they may willingly test-teach the material, and do so under favorable conditions.

► Time is a necessary and inflexible element in any professional betterment project. Among the aspects to be considered are 1) the total amount of time to be allocated, 2) the sub-allocation of time to group work, to individual work, etc. 3) the calendaring of time, and 4) the impact of demands for teacher-time both in, and out, of school.

During the first year, teaching a new program in mathematics is a definite increase in workload for a teacher. The increase may equal or exceed two days per month. How this time is scheduled greatly affects teachers' willingness to tolerate the increase. Ingenuous adjustments have not removed the requirement for time, but they have made the demand more tolerable.

Time is needed prior to the first session with pupils. In this regard successful programs have wide variations; some have had as little as 2 days, while others have had a series of 20-30 meetings spread over as many weeks.

Consultant-teacher sessions frequently continue with classroom instruction; necessarily so, where pre-classroom sessions have been brief.

This practice has been followed in SMSG Centers. Instructional concerns such as concept development, time allocation and adequacy of teacher background are more evident in this arrangement. Weekly sessions are most common. Wider spacing may be compensated by longer sessions, but cannot compensate for the delay in resolving teacher-problems.

► When the goal is immediate use of new instructional materials, those materials are the main ones for the in-service sessions. SMSG provides an extensive teachers' commentary to accompany each text.

► Where immediate classroom instruction is not involved, other materials may be more effective in developing mathematical background. Representative of this kind of material are the *Studies in Mathematics* that are briefly annotated on page 10 and more fully treated in Newsletters, Nos. 6 and 11.

Nearly every situation has unique aspects that have been utilized to increase the effectiveness of in-service education. Television has been used 1) for classroom instruction with each teacher as an observer along with the pupils and 2) for instruction directed to teachers.

Since there is a wide difference between development of concepts and memorization of statements and procedures, methodology is always an important part of teachers' professional study.

► To summarize, SMSG has obtained numerous reports on projects for introducing new instructional material into schools. From these the conclusion is clear that additional teacher preparation is essential; the mathematics to be taught is the focus of this preparation.

The four elements are to be judiciously selected:

1. Resource personnel competent in mathematics and mathematics teaching.
2. Teachers who desire to provide more effective instruction in mathematics.
3. Time sufficient in amount and scheduled

to achieve teacher learning prior to the time of need.

4. Instructional materials that indicate the teachers' classroom task and related materials that expand the mathematical topics.

With these elements a coordinator and his advisory resources can establish a workable in-service education program, and keep it adjusted to the the maximum benefit of the teachers involved.

ANNOUNCEMENT OF TEXTBOOKS FOR CLASSROOM USE

In keeping with its responsibility for developing and making available new textbooks, SMSG announces that the books listed below may be provided for general classroom use in the 1962-1963 academic year.

Newsletter No. 11 contains descriptions of the preliminary 1961 editions of these textbooks; it also provides information so the texts and commentaries may be ordered for inspection and study.

Revised versions are offered for September delivery; revision being based upon the test-teaching experience accumulated during 1961-62.

ADDITIONAL TEXTBOOKS AVAILABLE

MATHEMATICS FOR THE ELEMENTARY SCHOOL

- Grade 4, Textbook, (Parts 1, 2 and 3) $2.00
- Grade 4, Teachers' Commentary, (Parts 1, 2 and 3) $2.00
- Grade 5, Textbook, (Parts 1, 2 and 3) $2.00
- Grade 5, Teachers' Commentary, (Parts 1, 2 and 3) $2.00
- Grade 6, Textbook, (Parts 1, 2 and 3) $2.00
- Grade 6, Teachers' Commentary, (Parts 1, 2 and 3) $2.00
- Selected Units, Grade 4, E-4150 .... $ .75

INTRODUCTION TO SECONDARY SCHOOL MATHEMATICS

- Volume I, Textbook, (Parts 1 and 2) $2.00
- Volume I, Teachers' Commentary, (Parts 1 and 2) $2.00
- Volume II, Textbook, (Parts 1 and 2) $2.00
- Volume II, Teachers' Commentary, (Parts 1 and 2) $2.00

INTRODUCTION TO ALGEBRA

- Textbook, (Parts 1, 2, 3 and 4) $2.50
- Teachers' Commentary, (Parts 1, 2, 3 and 4) ... $2.50

GEOMETRY WITH COORDINATES

- Textbook, (Parts 1, 2 and 3) $4.00
- Teachers' Commentary, (Parts 1, 2 and 3) $4.00

(An Order Form will be found on inside back cover)

Observing the directions enumerated below will greatly aid the filling of orders:

1. June 10, 1962 is the deadline for orders to be delivered for the September opening of school. Mark orders **For September Delivery.**
2. Send orders to:

   A. C. Vroman, Inc.
   367 South Pasadena Avenue
   Pasadena, California
3. A classroom set is a minimum order for September delivery.
4. The list shows "ceiling" prices; invoices will be sent after delivery has been completed on all parts.
5. These texts are priced at cost; there is no discount.
6. Direct all inquires of an editorial nature to the SMSG office at Stanford University.

MATHEMATICS FOR THE ELEMENTARY SCHOOL, SELECTED UNITS. E-4150

Three selected units from Grade 4, EA-101—Concept of Sets, EA-105 – Sets of Points, and EA-110 – Concept of Fractional Numbers are collected in this single volume. It has two specific uses: 1) to use with Grade 5 textbooks as prerequisite content for that course and 2) to augment instruction in grades 4 or 5.

Mathematics for the Elementary School
Selected Units, Grade 4, E-4150 ...... $ .75
(Available June 1, 1962)

STUDIES IN MATHEMATICS

This is a series of publications which SMSG has generated for teacher education. They are being used in a variety of arrangements, particularly where considerable time is given to teacher education prior to initiating use of new instructional materials with pupils. Although they are described in SMSG Newsletter Nos. 6 and 11, their usefulness as in-service material for mathematics education warrants a complete listing here.

Some Basic Mathematical Concepts, by R. D. Luce. Elementary set theory with illustrations relating set concepts to various parts of mathematics .....SM-1 $ .80

Euclidean Geometry Based on Ruler and Protractor Axioms, by C. W. Curtis, P. H. Daus and R. J. Walker. Presents, for high school teachers, the approach to Euclidean Geometry used in the SMSG *Geometry* text ....SM-2 $ .90

Structure of Elementary Algebra, by Vincent H. Haag. Expands the approach to algebra found in the SMSG *First Course in Algebra* SM-3 $1.40

Geometry, by B. V. Kutuzov. Translation of a Russian text; useful as a source of supplementary material. SM-4 $2.75

Concepts of Informal Geometry, by Richard D. Anderson. A study of basic ideas, concepts and points of views of geometry; intended for junior high school teachers. SM-5 $1.45

Number Systems. A study of the structure of number systems intended to provide background for teachers of elementary school mathematics SM-6 $2.40

Intuitive Geometry. A companion volume to *Number Systems* to help elementary teachers develop subject matter competence in geometry SM-7 $1.25

Concepts of Algebra. A study with numerous exercises to provide elementary school teachers with understandings of algebraic concepts and language SM-8 $2.40

CONFERENCE REPORTS AS SUMMARIES

Four conference reports have been published by SMSG as a portion of its work in developing sample texts. These reports have been found to be a brief and efficient means of gaining information about the textbooks with which the several conferences were concerned. With one exception, the conferences were conducted by members of the writing teams and served two purposes:

1. To inform teachers, supervisors and consultants of SMSG Centers about the content of the courses and,
2. To provide insight into the spirit and rationale for teaching the newer content.

The conference proceedings are sufficiently detailed to provide a reasonable overview of the instructional materials to which they are related.

These conferences were planned as a concentrated form of in-service education. As such they complement the *Studies in Mathematics* and the teachers' commentaries. They contain questions and discussion of questions that were raised. The following briefs indicate the general nature of each conference report.

CONFERENCE REPORTS

Report of a Conference on Elementary School Mathematics, CR-1, $ .40.

On February 14-19, 1959, 64 mathematicians, psychologists, teachers and representative of scientific and governmental organizations attended the first SMSG conference. Discussions concerned elementary school mathematics, its curriculum, related teacher training, and research in teaching. Also included are reports on newer techniques and topics in school mathematics, the applications of psychology, and recommendations for a comprehensive study of the mathematics curriculum.

Report of an Orientation Conference for SMSG Experimental Centers, CR-2, $1.00.

This conference on September 19, 1959 surveyed the textbooks for each grade 7 to 12. These were the preliminary editions of *Mathematics for Junior High School* and *Mathematics for High School*. It reports the first concentrated in-service education session held for teachers who initiated use of these new textbooks.

Report of an Orientation Conference for SMSG Elementary School Experimental Centers, CR-3, $1.00.

These sessions on September 23-24, 1960 were devoted to the background and philosophy of *Mathematics for the Elementary School*, Grades 4, 5 and 6, as presented by the writing team for those textbooks. Particular attention is given to the unique topics to be incorporated into elementary school mathematics. Questions raised by teachers are answered.

Report of an Orientation Conference for SMSG *Geometry With Coordinates*, CR-4, $ .40.

The staffs that were to teach *Geometry With Coordinates* were assembled on September 23, 1961 to hear from the writing team of that textbook about its unique topics and about its similarities to other textbooks. The report summarizes talks on measurement, betweenness, coordinates, vectors, and geometry as a logical system. Attention was given to pedagogy as well as to content.

An Order Form including these four conference reports, CR-1, CR-2, CR-3, and CR-4 will be found on page 13.

IMPORTANT ORDER INFORMATION

Avoid error and disappointment by knowing how to order SMSG publications.

SMSG is engaged in an effort to improve mathematics instruction. This results in a rapidly changing list of publications – there are additions and deletions. The SMSG Newsletters give full current information as concisely as possible. We are not in competition with regular publishers of textbooks and do not follow their practices and services in the distribution of publications.

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6. There is no duplication of supplier for any SMSG publication.
   a. Those textbooks available from Yale University Press (SMSG, 92A Yale Station, New Haven, Conn.) may not be obtained anywhere else.
   b. Likewise monographs in the New Mathematical Library may be secured from:

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SMSG TEXTBOOK ORDER FORM — CLASSROOM QUANTITIES

Available Only From: A. C. VROMAN, INC.
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For textbooks to be published and distributed in the limited time allowed, we find it necessary to observe the following conditions:

1. June 10, 1962 is the deadline for orders to be delivered as school opens.
2. Mark orders "For September Delivery".
3. For September delivery, a classroom set is the minimum order.
4. The prices given here are "ceiling" prices; invoices will be sent after delivery has been completed on all parts. These are distributed at cost, no discount allowed.
5. INVOICE ADDRESS

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6. Give Shipping Address if different.

SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 15

April 1963

Reports

CONTENTS

Financial Support of the School Mathematics Study Group has been provided by the National Science Foundation

The two reports which follow are reminiscent of those included in SMSG Newsletter No. 10. Both indicate that students in SMSG classes do just as well on standard tests of mathematical skills as do students in conventional courses. At the same time, the SMSG students are exposed to and learn something about a number of concepts not available in conventional courses.

STUDENT ACHIEVEMENT IN SMSG CLASSES, GRADES 4 AND 5

(Summary of a report prepared by J. Fred Weaver)

During the 1961-62 school year each Elementary Center† conducted a testing program involving those children using the SMSG *Mathematics for the Elementary School* sample texts for grades 4, 5 and 6. The tests and inventories included in this program are summarized in Table I.

TABLE I
Elementary Center Testing Program, Grades 4-6, 1961-62

Test or Inventory	Grade 4 Fall 1961	Grade 4 Sprg. 1962	Grade 5 Fall 1961	Grade 5 Sprg. 1962	Grade 6 Fall 1961	Grade 6 Sprg. 1962
SRA Primary Mental Abilities (for ages 7 to 11)	x		x		x	
SRA Arithmetic Achievement Test (for grades 4-6, Form A)	x	x	x	x	x	x
SMSG Mathematics Test (*)		x		x		
SMSG Ideas and Preferences Inventory (**)	x	x	x	x	x	x

* Separate 4th and 5th grade tests, with content selected from the kind included in the SMSG sample texts, were developed and pre-tested prior to use. No such test was prepared and used in grade 6, since no children at this level had studied the full three-year sequence, grades 4-6.

** The same inventory, developed and pre-tested for this purpose, was used in all three grades, both fall and spring.

The present report is based on data derived from the testing program in grades 4 and 5. Furthermore, analyses and interpretations are based on a sample drawn from the total population of approximately 600 pupils in grade 4 and 1200 pupils in grade 5. The number of pupils included in this sample is indicated in Table II.

TABLE II
Number of Pupils in Sample

	Grade 4	Grade 5
Number of boys	112	111
Number of girls	101	83
Total number	213	194

† Preliminary versions of the SMSG texts for grades 4, 5, and 6, were tried out in classrooms in eight "Centers." Feedback information from these centers provided the bases for revision of the texts in the summer of 1962. Each center consisted of approximately a dozen teachers, a chairman, and a consultant.

CHARACTERISTICS OF SAMPLE

Performance on the SRA Primary Mental Abilities test may be expressed in various ways. Two such measures are: (1) the "Arithmetic Aptitude Quotient" [the ratio of a child's estimated "arithmetic age" to his chronological age], and (2) the "IQ Estimate." This information, along with information regarding chronological age, is summarized for the sample in Table III.

TABLE III
Selected Characteristics of Sample

	C.A. in months*		Arith. Aptitude Quotient		IQ Estimate	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
Grade 4: Boys	113.6	4.8	111.1	12.6	115.0	15.6
Grade 4: Girls	111.9	4.1	113.5	11.6	119.8	14.0
Grade 4: Total	112.8	4.6	112.2	12.2	117.3	15.0
Grade 5: Boys	125.1	6.5	110.9	13.3	116.2	15.0
Grade 5: Girls	123.2	4.7	110.5	13.9	116.2	14.1
Grade 6: Total	124.3	5.9	110.7	13.6	116.2	14.6

* As of approximately October 1, 1961

On the average, the 4th grade pupils were approximately 9 years 5 months old [in the fall]; the 5th grade pupils, approximately 10 years 4 months old. In terms of the entrance-age requirements in effect in many schools, the children in the sample would be of "normal" age for the grade involved.

Rather clearly, the children in the sample are "above average" in terms of both the index of arithmetic aptitude and the estimated IQ. To the extent that the sample is a valid one, this same observation would apply to the population from which the sample was drawn.

The differences in means between boys and girls have not been tested for statistical significance, but do not appear to be of particular consequence for purposes of this report.

ARITHMETIC ACHIEVEMENT STATUS

Based on scores from the two administrations of the SRA Arithmetic Achievement Test, Table IV summarizes the fall status, the spring status, and the resulting change in status of pupils in the sample on each of two factors: Reasoning [Problem Solving], and Computation.

[*Note.* One section of the SRA Arithmetic Achievement Test is entitled "Concepts," which clearly is a misnomer. This section involves the reading of numerals and selected vocabulary items (e.g., "multiplier," rather than "factor" as used in the SMSG sample texts). The scores on this section of the test have been disregarded for this and other reasons.]

The means reported in Table IV are derived from raw scores [i.e., number of rights]. The reported Grade Equivalents are approximated from the mean raw scores.

TABLE IV
Arithmetic Achievement Status

	Boys		Girls		Total	
	Mean Rights	Approx. Grade Equiv.	Mean Rights	Approx. Grade Equiv.	Mean Rights	Approx. Grade Equiv.
Grade 4						
Reasoning						
Fall Testing	18.4	4.2	20.0	4.4	19.1	4.3
Spring Testing	26.8	5.6	28.3	5.8	27.5	5.7
Change (Gain)	8.3	1.4*	8.3	1.4*	8.3	1.4*
Computation						
Fall Testing	10.2	4.3	9.4	4.2	9.8	4.3
Spring Testing	17.0	5.1	17.8	5.2	17.3	5.1
Change (Gain)	6.9	0.8*	8.5	1.0*	7.7	0.8*
Grade 5						
Reasoning						
Fall Testing	25.8	5.5	26.0	5.5	25.9	5.5
Spring Testing	32.8	6.5	33.0	6.5	32.8	6.5
Change (Gain)	7.0	1.0*	7.0	1.0*	7.0	1.0*
Computation						
Fall Testing	17.0	5.1	15.6	5.0	16.4	5.0
Spring Testing	24.9	6.0	24.3	5.9	24.6	5.9
Change (Gain)	8.0	0.9*	8.7	0.9*	8.3	0.9*

* This is the difference in approximate grade equivalents, not the approximate grade equivalent of the difference in mean rights.

In terms of the approximated Grade Equivalents reported in Table IV, it may be said for each subsample [4th grade boys, 4th grade girls, 4th grade total, 5th grade boys, 5th grade girls and 5th grade total] that in both Reasoning [Problem Solving] and Computation:

1. The mean rights was equal to or greater than the "grade norm" at the time of fall testing.

2. The mean rights was equal to or greater than the "grade norm" at the time of spring testing.

Furthermore, when it is recognized that approximately eight months elapsed between the fall and spring testings, it also may be said that in both Reasoning [Problem Solving] and Computation:

3. The mean gains equaled or exceeded "normally expected gains" in terms of Grade Equivalents.

In this connection note that the gain in computation was approximately the same for the 4th and 5th grades. However, in reasoning [problem solving] a greater gain was observed in the 4th grade than in the 5th grade. No pattern of significant sex difference in artithmetic achievement was observed at either grade level.

In general we may assert that in terms of the test administered and its grade norms, the use of the SMSG sample texts in grades 4 and 5 did not inhibit the mean arithmetic achievement of the pupils in the sample.

The mean gains reported in Table IV were the actually observed gains. It may be of interest to note how these gains compare with computed "regressed gains," based on predicted spring scores derived from an appropriate regression equation which takes into consideration the factor of test [or sub-test] reliability.

TABLE V

Observed versus Regressed Gains

	Mean Gain in Rights Observed	Regressed
Grade 4, Reasoning: Boys	8.3	8.1
Grade 4, Reasoning: Girls	8.3	8.8
Grade 4, Reasoning: Total	8.3	8.4
Grade 4, Computation: Boys	6.9	6.9
Grade 4, Computation: Girls	8.5	8.1
Grade 4, Computation: Total	7.7	7.5
Grade 5, Reasoning: Boys	7.0	6.6
Grade 5, Reasoning: Girls	7.0	7.0
Grade 5, Reasoning: Total	7.0	6.8
Grade 5, Computation: Boys	8.0	7.8
Grade 5, Computation: Girls	8.7	8.5
Grade 5, Computation: Total	8.3	8.1

Table V clearly reveals substantial degrees of agreement between observed and regressed mean gains in rights on the SRA Arithmetic Achievement test from fall to spring testings.

SMSG ACHIEVEMENT STATUS

Two special tests were developed to measure achievement in selected aspects of content emphasized in the 4th and 5th grade SMSG sample texts. These tests were entitled:

Mathematics for the Elementary School, Test A [Grade 4]

Mathematics for the Elementary School, Test B [Grade 5]

Each test consisted of these three parts:

I. Number and Operation
[Maximum raw score (rights): 55]

II. Geometry
[Maximum raw score (rights): 20]

III. Applications
[Maximum raw score (rights): 10]

[Maximum total raw score (rights): 85]

Table VI presents the means and standard deviations of distributions of raw scores on these tests, administered in late May at the end of the school year.

TABLE VI

Normative Data for 4th and 5th Grade SMSG Tests

		Grade 4 Mean Right	S.D.	Grade 5 Mean Right	S.D.
Part I:	Boys	30.5	9.8	28.5	8.4
Part I:	Girls	31.5	11.0	26.3	8.3
Part I:	Total	31.0	10.4	27.6	8.4
Part II:	Boys	11.2	3.1	12.0	3.6
Part II:	Girls	12.0	3.6	11.4	3.3
Part II:	Total	11.6	3.3	11.7	3.5
Part III:	Boys	4.1	2.5	3.7	2.7
Part III:	Girls	5.0	2.6	3.0	2.3
Part III:	Total	4.5	2.6	3.4	2.6
Whole Test:	Boys	45.9	12.7	43.8	13.0
Whole Test:	Girls	48.4	15.1	40.7	11.9
Whole Test:	Total	47.1	14.0	42.5	12.6

The performances reflected in the data of Table VI cannot be judged "good," or "bad," or otherwise. The data are normative data and must be interpreted only as such.

In Table VI a consistent pattern is observed in the differences between mean rights for boys and mean rights for girls. For grade 4, the difference between means consistently favors the girls; for grade 5, the difference between means consistently favors the boys. The reliability or statistical significance of these differences has not been tested to date, however.

ATTITUDINAL FACTORS

A special instrument, the "Ideas and Preferences Inventory," was devised and used in both grades 4 and 5 in an attempt to measure attitudinal factors. The test was administered in the fall and again in the spring.

Factor analysis of a preliminary form of the instrument resulted in the identification of nine sub-scales.

Analysis of the fall and spring scores indicates that the following observations are probably valid:

1. With the exception of three sub-scales, the mean fall testing scores indicated a tendency — but not a marked one — in the direction of favorable attitudes toward mathematics. This was true for both boys and girls in both the 4th and 5th grades. In the case of the three sub-scales, there was a slight tendency in the direction of unfavorable attitudes toward mathematics.

2. The mean fall testing scores for 5th grade pupils — who already had used the SMSG sample texts in grade 4 — are indicative of no more favorable attitude toward mathematics than are the mean fall-testing scores of 4th grade pupils — who were using SMSG sample texts for the first time.

3. The observations just reported do not reinforce the oft heard cries regarding the decidedly negative attitudes toward arithmetic on the part of intermediate-grade children.

4. Both boys and girls, in both 4th and 5th grades, showed a slight — but far from marked — tendency to have more favorable attitudes toward mathematics at the end of the year than at the beginning of the year. [This is a general tendency, not without some exception.]

The data derived from administrations of the SMSG Ideas and Preferences Inventory give rise to the following question: Was the actual change in attitude toward mathematics as little as that generally observed, or was the insttument insensitive to more marked attitudinal changes that really did take place? Informally reported reactions from many teachers would tend to support the latter hypothesis [or fail to support the former hypothesis], but we have no evidence at hand to give an answer with reasonable confidence.

Evaluation of SMSG Text — Grade 4

(Summary of a report from the Minnesota National Laboratory)

I. Description of the Experiment

Ten pairs of 4th grade classes in the vicinity of Minneapolis and St. Paul were chosen to participate in this evaluation. In each pair one class was designated as the experimental class and the other as the control. The two classes in each pair were taught by two different teachers. Six of the pairs were each from the same school. Each of the other four were from different, but comparable schools.

In all pairs of classes, with one exception, the students in the two classes were matched on the basis of I.Q. and achievement and, to some extent, by the teacher's difference between the experimental and the control classes. The two pairs which show the maximum difference each way had the following I.Q.'s:

	E (Exp. Class)	C (Control Class)	E - C
Pair 8	Av. I.Q. = 116	Av. I.Q. = 104	12
Pair 10	Av. I.Q. = 107	Av. I.Q. = 113	— 6

The values of E - C for the other eight pairs of classes were: —1, 0, +7, —1, 0, —1, +4, +2.

With respect to the ten pairs of teachers chosen for the experiment, it is more difficult to make a comparison of teaching ability. A record of educational background and teaching experience was obtained for each of the twenty teachers. There was some indication that the experimental teachers had a slightly better background in mathematics; however, the difference between the two groups in this regard was not susceptible to any quantitative measurement. However, the average salary of the experimental teacher was $7054 compared with $5490 for the control teacher.

During the past year (1961-62) the ten experimental classes have been taught from the SMSG 4th grade texts, while the control classes were taught from conventional texts. To compare the progress of the students in the two groups throughout the year, the STEP Test 4A was given to all twenty classes in September, 1961 and again in the first week of May, 1962. They were also given the DAT (Differential Aptitude Test) in February, 1962.

II. Statistical Data

The means for the entire experimental and the entire control group on the STEP and DAT Tests are given in the table below.

	DAT	X_1 STEP 4A Sept.	X_2 STEP 4A May	$X_2—X_1$
E	252.7	241.7	249.4	7.7
C	254.1	242.4	249.6	7.2

The following table gives a comparison of relevant data for each of the ten pairs of classes. The test scores shown are mean scores.

Pair	Class	I.Q. Range	Average I.Q.	DAT	X_1 STEP 4A Sept.	X_2 STEP 4A May	$X_2—X_1$
1	E	80-132	110	250.9	239.6	243.3	3.7
	C	88-129	111	252.4	239.9	245.8	5.9
2	E	94-125	111	252.7	241.8	250.0	8.2
	C	71-128	111	254.7	241.3	248.9	7.6
3	E	97-123	111	252.6	241.3	250.6	9.3
	C	84-127	104	251.3	242.7	249.9	7.2
4	E	86-139	115	254.1	243.9	251.9	8.0
	C	96-137	116	254.4	242.6	249.4	6.8
5	E	76-129	110	254.3	242.3	250.2	7.9
	C	73-139	110	254.5	240.7	251.4	10.7
6	E	85-130	110	252.2	244.4	252.0	7.6
	C	83-141	111	255.6	247.5	253.5	6.0
7	E	79-126	109	251.2	240.6	246.5	5.9
	C	94-125	105	250.9	241.1	245.1	4.0
8	E	85-147	116	255.4	242.8	254.1	11.3
	C	75-137	104	254.8	240.4	250.3	9.9
9	E	94-121	109	251.6	240.5	248.7	8.2
	C	83-124	107	256.8	245.1	251.9	6.8
10	E	73-133	107	251.7	238.9	245.6	6.7
	C	98-137	113	255.3	242.4	248.6	6.2

E is used for "Experimental".
C is used for "Control."

The range of standard deviations in the STEP Test scores X_1 and X_2 was from 7.3 to 11.9 for the twenty classes; that for X_1 and X_2 was from 4 to 7.

The standard deviations for the totals were:

	X_1 STEP 4A Sept.	X_2 STEP 4A May	X_2-X_1
E	8.6	10.2	6.0
C	9.6	10.2	6.0

On the basis of the above data it would appear that there was no significant difference in the progress of the two groups (experimental and control) in traditional arithmetic over the school year. The slight measureable difference which did exist was in favor of the experimental group.

III. CONCLUSION

It is believed by those who have participated in it that the experiment has been successful and gives evidence for the superiority of materials such as SMSG over traditional texts. As pointed out earlier there is no significant difference in the progress of the two groups in mastering traditional work. However, the experimental group has been exposed to a number of basic mathematical concepts which the control group has not. Also, the experimental group has spent considerable time on units (e.g. sets, geometry) which is not reflected in their performance on STEP 4A.

The present experiment is based on too small a sample and is spread over too short a time interval to be conclusive. Whether or not the SMSG approach gives a more lasting mastery of the techniques of arithmetic, by being based on more thorough understanding, can be finally determined only by continued experimentation and evaluation over longer periods of time and with larger numbers of classes.

The SMSG high school texts were written for college capable students. However, many of the writers felt that the kind of mathematics in these texts might well be appropriate for a wider audience. Those who prepared the junior high school texts shared this feeling. Accordingly, an attempt was made to test this hypothesis.

Two publications were prepared. The first, "Introduction to Secondary School Mathematics", covers the material in "Mathematics for Junior High School" (Volume 1 and part of Volume 2). The second, "Introduction to Algebra", covers the material in "First Course in Algebra". These texts are described in Newsletter No. 11.

In preparing these revisions, the authors made an effort to include all the mathematics in the original texts, although presenting it in a different style. That the change in style did not result in the loss of any of the mathematics is demonstrated by the following report.

Comparison Experiment of SMSG 7M and 9M Texts with SMSG 7th and 9th Grade Texts

(Summary of a report submitted by Walter Fleming for the Minnesota National Laboratory)

I. DESCRIPTION OF THE EXPERIMENT

Sixteen Minnesota schools participated in this experiment which involved fourteen seventh grade classes and eighteen ninth grade classes. Each of the sixteen teachers involved in the experiment taught one experimental class with the SMSG "M" text (the members of the writing teams which prepared "Introduction to Secondary School Mathematics" and "Introduction to Algebra" came to refer to these as "M" texts) and one experimental class with the regular SMSG text for a given grade level. The teachers covered essentially the same topics in each of the two experimental classes and gave the same tests to both classes. The same length of time was spent on each unit. This was accomplished by using enrichment material from time to time in one class or the other. As far as it was possible the teachers controlled other classroom variables such as motivation, teaching aids and assignments. The purpose of the experiment was to identify the difference in mathematics achievement, if any, due to the use of different SMSG texts.

Each of the seven teachers at the 7th grade level and each of the nine teachers at the 9th grade level had at least one year of previous teaching experience with the SMSG texts, either regular or "M". All of the teachers except one had participated in summer institutes or academic year institutes in which SMSG materials were used.

2. DESCRIPTION OF CLASSES

The classes involved in this experiment were very nearly uniform in size, the average being about thirty students per class. Both experimental classes in each school were taught by the same teacher and the class periods for the two classes were of equal length. The seventh grade students were drawn from the upper half of each school's seventh grade students. The ninth grade students were drawn from the upper third of the ninth grade students of the school.

In each school the students were assigned to the experimental classes in such a way that the two classes were approximately of equivalent ability. Assignment was made on the basis of I.Q. tests or other ability tests, and teacher judgment. The following tables show the I.Q. range and the mean I.Q. (or grade level) for each of the fourteen seventh grade classes and for each of the eighteen ninth grade classes.

Ability Ranges of 7th Grade Classes

School	Ability Test	Text Book Used	Low	Median	High
1	Lorge-Thorndike	R	101	116	129
	Verbal I.Q.	M	95	115	135
2	Lorge-Thorndike	R	79	109	124
	Verbal I.Q.	M	99	113	139
3	Iowa Basic Skills	R	5.1	7.1	8.4
	(Grade Level at end of 6th Grade)	M	5.1	7.1	8.4
4	Otis (Beta)	R	101	109	130
	I.Q.	M	103	109	126
5	Kuhlman-Finch	R	109	123	140
	I.Q.	M	103	120	140+
6	Otis (Beta)	R	101	116	134
	I.Q.	M	101	115	134
7	Otis (Beta)	R	91	114	136
	I.Q.	M	89	109	139

Ability Ranges of 9th Grade Classes

School	Ability Test	Text Book Used	Low	Median	High
1		R			
		M			
2	Lorge-Thorndike	R	104	121	141
	Verbal I.Q.	M	106	121	138
3	Otis	R	101	114	135
	I.Q.	M	90	117	138
4	Otis	R	116	126	144
	I.Q.	M	107	120	129
5	Otis	R	99	113	133
	I.Q.	M	101	115	126
6	Lorge-Thorndike	R	102	124	147
	Verbal I.Q.	M	102	124	147
7	Otis (Beta))	R	94	107	121
	I.Q.	M	83	103	123
8	Lorge-Thorndike	R	86	113	129
	Verbal I.Q.	M	94	115	137
9	Lorge-Thorndike	R	90	113	132
	Verbal I.Q.	M	91	115	134

3. AMOUNT OF MATERIAL COVERED

(a) 7th grade

On the average the amount of material covered in class was that which is contained in the first three parts of the "M" text. The teachers felt that the material in the first three parts of the "M" text constitutes a good years work, even for good students.

(b) 9th grade

There is considerable variation in the number of chapters covered in the nine schools. Each teacher however, covered approximately the same amount of material in his two classes. In some cases this did not go beyond Chapter 12 of the "M" text. One teacher managed to cover all of the material in the "M" text. On the average the material covered in both classes amounted to that contained in the first fifteen chapters of the "M" text.

4. NUMBER OF SMSG UNIT TESTS ADMINISTERED

(a) 7th Grade

One teacher administered all eight of the SMSG unit tests. (These were special tests, each covering the material in two or three chapters, used in the classroom evaluation of the preliminary versions of the "M" texts.) She explained, however, that she had not covered the material on which the last three tests were based. Another teacher gave only the first five of the tests. The other five teachers gave the first six of the tests. This means that on the average both classes were tested on the material included in the first fifteen chapters of the 7M text.

(b) 9th Grade

Only one teacher administered all seven of the SMSG unit tests. Two teachers gave the first six tests, four others gave only the first five tests, while the other two gave only the first four tests. The test results from one school were not available at the time when this report was prepared. Consequently, the arithmetic means in the table in part (b) of the next section are based on the test scores from eight schools (instead of nine), except for Test V in which the means are based on the scores from six schools.

5. SUMMARY OF TEST SCORES ON SMSG UNIT TESTS I THROUGH V

(a) 7th Grade

		Mean High Score	Mean Low Score	Mean of Medians
I	Reg.	32.6	10.0	22.4
	"M"	34.1	11.1	24.7
II	Reg.	31.9	10.0	23.7
	"M"	32.6	13.0	25.3
III	Reg.	31.7	7.4	19.9
	"M"	31.7	9.0	21.1
IV	Reg.	27.3	4.7	16.0
	"M"	26 7	6.6	18.3
V	Reg.	25.0	8.0	17.3
	"M"	26.9	7.6	19.3

(b) 9th Grade

		Mean High Score	Mean Low Score	Mean of Medians
I	Reg.	33.9	19.4	28.5
	"M"	34.7	19.4	28.9
II	Reg.	33.7	14.4	24.7
	"M"	34.6	16.7	26.4
III	Reg.	30.4	11.6	22.0
	"M"	30.9	12.4	23.1
IV	Reg.	26.4	9.5	16.7
	"M"	26.6	10.6	18.2
V	Reg.	24.7	8.0	15.8
	"M"	25.8	8.0	17.3

NOTE 1: In each test except Test II the possible score is 35; in Test II it is 36. Each teacher reported the high score, the low score and the mean score on each test for each of his two classes. In the table above, the arithmetic means of these scores are shown.

NOTE 2: In the 7th grade the mean of the medians for the "M" section is about 1.9 points higher than for the regular section. In the 9th grade the mean of the medians for the "M" section is about 1.2 points higher than for the regular section.

6. SUMMARIZATION OF STEP TEST DATA

The purpose of this study was to compare the mathematics achievement of high ability students using the "M" and "R" editions of seventh and ninth grade SMSG texts. The data on mean high score, mean low score, and mean median score reported in section 5 of this report gives this comparison with the SMSG unit tests. The comparisons of this section are with the scores on the STEP tests given in fall of 1961 as pre-tests and in the spring of 1962 as post-tests. The tables below this section report the mean and standard deviations for the classes combined within experimental status and grade. The same statistics are given for the seventh and ninth grades which are a part of the SMSG evaluation experiment continuing in the Minnesota National Laboratory for the fourth year. The data given is for the 1961-62 phase of that particular experiment.

Mean Scores

Grade 7 Comparison

	STEP Pre-Test	STEP Post-Test	Gain	Adjusted Post-Test	n
		Regular Text			
Means	265.2	273.4	8.2	273.6	198
St.D.	11.44	10.85	8.		
		"M" Text			
Means	265.9	274.4	8.5	274.1	189
St.D.	10.99	9.313	8.		

Mean Scores

Minn. National Laboratory Grade 7

	STEP Pre-Test	STEP Post-Test	Gain	Adjusted Post-Test	n
Means	265.8	273.8	8.0	273.6	216
St.D.	9.429	9.306	8.		

Mean Scores

9th Grade Comparison

	STEP Pre-Test	STEP Post-Test	Gain	Adjusted Post-Test	n
		Regular Text			
Means	282.2	285.8	3.6	285.7	268
St.D	11.3	9.799	8.		
		"M" Text			
Means	281.8	284.7	2.9	284.8	249
St.D.	10.37	9.81	8.		

Mean Scores

Minn. National Laboratory Grade 9

	STEP Pre-Test	STEP Post-Test	Gain	Adjusted Post-Test	n
Means	279.3	284.0	4.7	286.2	270
St.D.	10.9	11.84	8.		

Estimated regression equations, where x = STEP pre-test score, y = STEP post-test score, are

7—Reg.	$y = .701x + 87.5$	$r = .74$	S.E. = 7.3	n = 198
7—M	$y = .629x + 107.3$	$r = .74$	S.E. = 6.3	n = 189
7—Eval.	$y - .626x + 107.4$	$r = .63$	S.E. = 7.2	n = 216
9—Reg.	$y = .679x + .93.4$	$r = .72$	S.E. = 6.8	n = 249
9—M	$y = .600x + 116.6$	$r = .69$	S.E. = 7.1	n = 268
9—Eval.	$y = .809x + 58.1$	$r = .74$	S.E. — 7.9	n = 270

The standard deviation of the coefficient of x is approximately .04 in each case and the standard deviation of a difference of 2 coefficients is, accordingly, .056. The difference between the regular and "M" group regressions is within the expected experimental variability although the question of whether the slope is larger for the regular than for the "M" text is being further investigated within the pair of classes of each teacher. For the ninth grade the slope for the evaluation experimenr students is appreciab,y larger than those for the comparison experiment.

For the 7th grade the regression of the post-test on the SCAT pre-test, as well as STEP, has been compared. These are, where y = STEP post-test, x = STEP pre-test, n = SCAT — Total.

7—Reg.	$y = .489x + .300n + 60.4$	$R = .77$	S.E. = 7.0	n = 198
7—M	$y = .437x + .304n + 73.4$	$R = .78$	S.E. = 5.9	n = 189
7—Eval.	$y = .385x + .413n + 57.4$	$R = .70$	S.E. = 6.7	n = 216

The significance of the differences here is being investigated.

For the classes combined within each experimental status 95% confidence intervals, in the order statistics for the quartiles have been computed. These together with those of the STEP national norms for fall testing of the next higher grade are given on next page.

	Q_1	M	Q_3
7—Reg.	264-269	273-276	279-282
7—M	266-270	273-276	279-282
7—Eval.	266-270	273-275	277-282
STEP — Norms	251	261	269

	Q_1	M	Q_3
9—Reg.	277-282	284-287	290-294
9—M	277-282	284-286	289-291
9—Eval.	275-280	285-287	290-292
STEP — Norms	261	271	280

This comparison prompts the following comments. All three experimental groups are far above the national averages as given by ETS. However, the differences in STEP scores between the seventh and ninth grade for the experimental groups are about the same as the differences in the norms group. If one assumes that above average students can learn at a higher rate and that STEP is still a reasonable test of knowledge for well above average students, the indication is that the learning capacity of these students is not being filled.

The comparison between the 7th and 9th grades just made is not a comparison between SMSG and conventional curricula. Such a comparison was not intended in this experiment. Most of these students have been in SMSG classes one year. To the extent that SMSG does increase the rate of learning it would have done so for both groups and the differences would then be the same.

The effect of psychological variables on the learning of mathematics is clearly of importance to those wishing to improve the latter. An investigation of one aspect of this area, attitudes towards mathematics, has been carried out for SMSG in a project under the direction of Professor Richard Alpert, Harvard University. The following report summarizes the work done in this project.

Psychological Factors in Mathematics Education

(Summary of a report submitted by R. Alpert, G. Stellwagon, and D. Becker)

A Short Description of the Project

I. INTRODUCTION

This paper reports the completion of a major investigation of many of the psychological variables most relevant to seventh grade mathematics education. This study, begun late in 1959, includes not only measures of the students themselves, but of their pararents and teachers as well. The following diagram indicates the scope and general design of our study. On the right hand side of the figure are our dependent or criterion variables, while the left column contains those factors which contribute to, determine, or affect the criterion variables:

Independent Variables — *Dependent Variables*

Text
Teacher
Parents
Past Performance
Intellectual Ability
Peers
School Environment

— (Student)

Mathematics performance

Attitudes & Motives

own ability towards mathematics

Although considerable pilot work was carried out during 1959-60, the major study, discussed in this report, was carried out during the academic year 1960-61 and subsequently analyzed.

The student sample consisted of approximately 270 subjects, half boys and half girls, all essentially middle class and attending the public schools in a surburb of a large eastern city. Half of the sample were in SMSG classes, and the other half were in conventional mathematics classes. For the development of our teacher-instruments, to be described below, 20 teachers were used, while for the main study, 8 were tested, teaching a total of 12 different classes. In the parent-analysis, the parents of 40 boys and 40 girls, stratified on the basis of IQ and socio-economic class, were selected. Both parents were studied, giving a total of 160 men and women comprising the parent sample.

II. INSTRUMENTS

The following is a brief description of the instruments we used to assess the various variables with which we were concerned.

A. *Student variables*

1. *Mathematics-related attitudes*

The mathematics attitude-questionnaire is comprised of a number of separate scales which measure student attitudes toward education and school subjects, particularly mathematics. These scales were designed to ascertain, for example, how much the students value education; how much they prefer mathematics to other subjects; whether they think mathematics is easy or hard, fun or dull, practical or not, and so forth.

2. *Mathematics-related anxiety*

Anxiety scales were developed in order to measure anxiety specific to mathematics examinations. The test is scored on two scales: a debilitating scale and a facilitating scale. The former measures harmful (performance-reducing) anxiety; the latter measures the type of reaction to stress which aids performance.

3. *Feelings associated with mathematics* ("mathematics affect")

To allow students to express feelings which they would perhaps not express in an attitude-questionnaire (when they know they are talking about themselves), a "projective" measure of mathematics affect was developed. A set of pictures with mathematics-relevant cues was shown, and the students wrote brief stories about each picture. For example, from the story a child writes about a picture depicting two youngsters working out a mathematics problem at a blackboard, one can infer his feelings towards mathematics.

4. *Actual mathematics achievement and IQ*

For measuring mathematics achievement, two very straight-forward scores were used. One was a standardized "mathematics average" performance score on the Metropolitan Achievement Test, administered in both sixth and seventh grade: and the other was the student's mathematics grade in school for both sixth and seventh grade. The IQ scores were provided to us by the school system.

5. *Self-concept* (how one sees oneself)

The self-concept instrument is oriented towards tapping the child's view of his own degree of adequacy (a) generally, (b) in the school situation and (c) in a social environment.

6. *Levels of expectation and aspiration regarding school*

These measures were obtained by direct statements from students with regard to their expected next report card grade, how well they expected/wished to perform in relation to the rest of the class, and how far they expected/wished to go in their education.

B. *Parent variables*

An interview and a questionnaire were developed in order to determine the extent to which the values and attitudes of parents (a) are consonant with those of SMSG, and (b) determine a child's attitude toward mathematics. These instruments were designed to provide us with information on the following:

1. Value placed on mathematics by parents
2. Pressure on child for mathematics achievement
3. Parents' conception of mathematics
4. Parents' mathematics affect (feelings, both positive and negative, towards mathematics)
5. Parents' value on achievement
6. Level of aspiration for the child (e.g., how far they would like their child to go in school)
7. Parents' value on intellectual activity
8. Occupational and educational level of the parents.

C. *Teacher variables*

1. *Teacher Interview*

This interview was designed to gather data on variables such as the following:

1. Feelings, memories, and attitudes of the teacher vis a vis his own background in mathematics.
2. The conception of himself as a teacher, as a mathematician, and as a problem solver.
3. Attitudes towards students, mathematics, problem-solving, insight, the respective school system, discipline, and indivdiual differences among students.
4. Sensitivity or empathy toward the students and toward the various areas of mathematics.
5. The rewards of teaching mathematics which the teacher receives and particularly values.

2. *Interaction Sample: Teacher (IS-T)*

This instrument rates classroom interaction (teacher — student) either by time-sampling (scoring on various dimensions whatever behavior is occurring at certain fixed moments, e.g., every 60 seconds) or by scoring every verbal behavior of the teacher from a tape recording made of the class. The former method was the one used in this study.

3. *Molar Observations*

The Molar Observation Scales are adaptations and extensions of the Teacher Interview ratings so as to enable trained classroom observers to rate the teachers on a more molar level than is possible with the IS-T. This permits a check on whether the personality characteristics which the teacher exhibits in the interview are manifest in the classroom situation as well; for, indeed, it is here that the teacher is acting upon the student. Since the IS-T observers were required to spend two class periods observing each teacher, it was felt that they were qualified to rank each teacher according to the Molar Observation categories. Since the Molar Scales correlated remarkably highly with the Teacher Interview scores, the data from these two instruments were combined to form an extremely powerful and fruitful teacher-measure.

III. RESULTS

The results of a study of this nature are necessarily complex. In this report we can do little more than very briefly and selectively sample our results. For interested readers, the full report of the results will appear during the coming year in book form.

A. *Student variables and Performance*

It was found that significant correlations existed between Performance and all of the above-mentioned student variables except "mathematics affect." Thus, the following student variables are correlated positively with Performance:

- High mathematics-attitudes
- High mathematics facilitating anxiety
- Low mathematics debilitating anxiety
- High Self-concept (especially School Self-concept)
- High IQ
- High level of expectation

Interesting sex-differences should be noted; for both boys and girls, School Self-concept (how adequate the child perceives himself to be in coping with school) is correlated positively with Performance. However, the data show that the Self-concept scales are more intercorrelated for boys than for girls. This indicates that more of a boy's life is bound up with his performance in school. Thus, a girl's opinion of herself socially is not as closely related to her School Self-concept as is a boy's.

B. *Interrelationships of Student variables*

Here the picture is the same for both boys and girls. The most important student variables (anxiety, mathematics attitudes, self-concept, and level of expectation) are all strongly intercorrelated; these variables correlate not only with Performance, but also *among themselves.*

Past Performance → Level of Expectation →

Self-concept → anxiety
attitudes

The above diagram schematically depicts the theoretically crucial connections among the variables. These relations are deemed "crucial" because it is through them that we believe *change* can operate. It is impossible to describe completely this proposal in a few sentences, but a brief outline may suffice: The child's past performance, and specifically his perception of that performance, leads him to form a level of expectation regarding his future performance. As recent research has shown, expectancy has a powerful determinant effect upon future performance. Thus, a self-perpetuating cycle of level of expectation → performance → level of expectation is formed. This then affects the child's self-concept, which is integrally related to his attiutdes and anxiety. If the Past Performance → level of expectation link can be broken, radical changes in performance may result. Although proposals for how this may be accomplished are beyond the scope of this report, experimental programs which help the student break set (change his expectancy) would seem to be warranted.

C. *Parent variables and Student Math attitudes*

Several variables from the Parent interview were found to correlate significantly with some student variables. Again it was the case that the sex of the child was shown to be important or not important, depending on the variables. Obviously only a small portion of the results can be mentioned here.

For both boys and girls, favorable student attitudes toward mathematics were positively related to the extent of mathematics education desired for the child by the parents; for boys alone were favorable attitudes toward mathematics positively correlated with parental importance placed on grades, as well as with parental demands for high grades. It is interesting to note further that, for girls, favorable attitudes towards mathematics were *negatively* correlated with parental importance placed on grades.

For both boys and girls, favorable student attitudes toward mathematics were positively correlated with two "competition scales" of parents: (a) competition viewed by parents as necessary in the modern world; and (b) competition evaluated by parents as "good."

An interesting cross-over between the sexes was found between favorable student attitudes toward mathematics and two "educational goals" parent-variables, as diagrammed below:

Parent's conception of the educational goals of a junior high school mathematics program

	positively related to mathematics attitudes	negatively related to mathematics attitudes
boys	to aid the intellectual development of the child	ability to deal competitively with practical everyday problems
girls	ability to deal competitively with practical everyday problems	to aid the intellectual development of the child

D. *Teacher-variables and Student-variables*

Anxiety and Affect are the two main student-variables that relate to the four clusters of teacher-variables which we labeled as

(a) High Theoretical Mathematics Interest (e.g., mathematics important as a logical system)
(b) High Psycho-social Concern (e.g., concern with the student as a psychological being; awareness of the role a teacher plays in molding the student personality-wise)
(c) Involvement in Teaching
(d) Personality Characteristics Cluster (e.g., "warmth," "patience," "little social distance" maintained between teacher and student, and others)

For boys, all four of the above clusters (or "factors") are correlated positively with Low Debilitating Anxiety, as was predicted by our staff. This is not the case for girls.

In regard to Affect, the data support the following conclusions: Boys respond with positive feelings toward mathematics to theoretically-oriented and involved teachers, regardless of the teacher's sex. However, the teacher's gender apparently determines whether boys or girls will respond with favorable feelings toward mathematics to psycho-social characteristics and the Personality Characteristics Cluster in the teachers. In general, it seems that the more "objective" factors (theoretical orientation and involvement in teaching) do not depend on the teacher's gender, while the more "subjective" or interpersonal factors (psycho-social concern and the Personality Characteristics Cluster) are effective only along same-sex lines.

E. *SMSG — non-SMSG Differences*

In this area, also, Affect and Attitudes will be the two student-variables discussed. The overall results indicate that the experimental program does not increase students' positive feelings toward mathematics, either absolutely or relative to the traditional mathematics program. There is clearly evidence, however, of powerful interaction between program and teacher. For example, in cases where a teacher teaches both SMSG and non-SMSG classes, a highly theoretical orientation on the part of the teacher leads to high positive affect in SMSG classes, but NOT in non-SMSG classes. In other words, it appears that a theoretical teacher's effect on positive mathematics affect is greatly facilitated when he is teaching a theoretical mathematics program. Thus, there are indications that the combination of a certain type of mathematics teacher with a certain mathematics program may generate results which are significantly stronger than the sum of the uncombined parts.

In regard to student attitudes toward mathematics, the results are central in evaluating the effects of the SMSG program. In the Fall, at the start of the school year, the SMSG students are found to be more favorably oriented toward mathematics than the non-SMSG students. However, the data show that, at the end of the school year, the SMSG program has not been able to maintain momentum. While non-SMSG mathematics attitudes remained relatively constant, SMSG students' attitudes fell. The general enthusiasm was gone. These results, while failing to show the hoped-for beneficial attitudinal effects of the experimental program, do show that it is indeed possible to arouse seventh graders' excitement about mathematics. This in itself is encouraging; the job now is to modify further the material and to *train teachers* in a way that will justify the students' initially high expectations.

IV. THE FUTURE

This study has found that in mathematics education there are significant and theoretically crucial relationships among teachers, seventh grade mathematics students, and parents — relationships which are measurable and modifiable. These results generally suggest:

1. the need for more attention by educators and writers of texts and supplementary material to those characteristics of the school experience which affect the psychological determinants of successful mathematics education;
2. the need for considerably more investment in teacher *selection* and *training*. Here we should consider the possibility of grouping students in relation to teacher-characteristics;
3. the urgent need for longitudinal research designed to demonstrate the patterns of mathematics performance and concomitant psychological variables emerging over time. Only through such studies can we truly grasp critical means-ends issues in mathematics education;
4. the need for further classroom experimentation, including, for example, self-instruction programs ("teaching machines"), the use of specially trained teachers, the use of innovative teaching techniques, and the development and use of training films;
5. the need for further development and refinement of those instruments designed to assess critical psychological and performance variables in mathematics education;
6. the need for a more careful consideration in course design of the meaning of mathematics education to women in our society;
7. the need for communicating to parents the nature of their impact on their children's mathematics education.

The more specific implications of our results will appear in the forthcoming detailed research volume. In addition, many of the specific results have been and will continue to be incorporated directly into the longitudinal study which is now underway.

In view of the pre-service programs for teacher preparation which have been available in the past, it is clear that most teachers, especially at the elementary level, need some in-service assistance in order to teach effectively the kind of mathematics in the SMSG sample texts. Discussions of in-service programs appeared in Newsletters Nos. 5 and 12.

In order to obtain additional information on in-service programs, especially for elementary school teachers, a study was conducted during the academic year 1961-62, a report of which follows. It is hoped that the observations contained in this report will be of assistance to schools interested in establishing new in-service programs or in improving existing ones.

A REPORT ON IN-SERVICE EDUCATION

(Prepared by George L. Roehr)

SOURCE OF THIS REPORT

During the 1961-1962 school year arrangements were made to accumulate some observations on locally devised in-service education programs. This report is based upon 15 different programs, located in ten different states. These programs involved approximately 185 teachers and 6500 pupils, and used the preliminary editions of *Mathematics for the Elementary School.* Also, these schools agreed to devise an in-service education program for the teachers using these texts and to provide SMSG with requested data relating to their in-service education progress in mathematics.

ADMINISTRATIVE DIRECTORS

For each location, there was a local staff person, usually a supervisor or administrator, who was mainly responsible for organizing and conducting the in-service education program. For the purpose of this report these 15 persons will be identified as "observers". Their local staff titles include assistant superintendent, assistant to the superintendent, director of elementary education, supervisor, etc. It was the "observers" that provided the variety of information requested concerning his in-service education project including an extensive terminal report.

For each of the 15 projects there were one or more "resource persons" who were capable of instructing the teachers in the mathematics they were to teach.

In brief then, in each situation there were four functional categories of persons; 1.) an administrative director who also served as our observer, 2.) the resource person(s), 3.) the participating teachers, and 4.) the pupils. While this report will stress "2" and "3", some remarks are essential concerning "1".

The administrative director guides the creation of the in-service education program; he needs the staff authority to obtain consideration of, and decisions upon, all institutional matters influencing the success of the venture. This is a central staff function and it is not unique for mathematics. However, it is the purpose of this report to expose those areas that require the attention of those whose function it is to serve as an administrative director. While no pattern or ritual assures a superior in-service education program, thoughtful use of available resources can produce beneficial results from a wide variety of circumstances.

PRIORITIES FOR TIME

Every in-service education project consumes varying amounts of time for different persons. For each instance where in-service education is deemed essential, it is rational to examine the allocation of teacher time between classroom work with pupils and study of curriculum materials. The problem might be stated thus, "Do the pupils benefit when 30 possible classroom hours per week are decreased to 28 by allocating 2 hours to in-service study of the mathematics being taught?" One project was in a unique school system having a general continuing 8% allocation of normal school week time devoted to in-service education. This time could be scheduled for the most pressing in-service educational needs.

In-service education projects are specialized educational activities subject to a multiplicity of factors that are varied in arrangement and degree. Hence the drawing of inflexible conclusions is not warranted but it should be useful to note facets for study along with guidelines to aid those who must use the in-service education resources available to their school system.

SIZE EFFECTS

What size of school systems made up our 15 projects? There were 27 school systems. Thirteen projects were each concerned with a single system, while two in California were directed by staff members from the county schools office and together accounted for 14 systems, six in one and eight in the other. Eleven of these were in the "small" or "medium" categories of districts as tabulated below.

The size of the districts are indicated by the following table:

Distribution of 27 School Systems In Three Size Categories by Enrollment

Small	Medium	Large
1000 or less	1001 to 3000	3000 and more
7	6	14

It is fairly evident that the existence of a central office staff is related to the existance of programs of in-service education though in the two California instances the administrative director function was located in the county schools office. Within a single system the time required to establish an in-service project for 20 teachers is little more than for five teachers.

The administrative director requires time in sizeable blocks to determine the general nature of the curriculum changes sought and the related in-service education. When this has been decided, other blocks of time will be consumed in making arrangements for the first and ensuing in-service sessions.

SINGLE OR MULTIPLE GRADE COVERAGE

Most of the projects were for teachers at grade four, others include teachers at grade four and five, still others had teachers at grades four, five, and six. In all there were 146 teachers at grade four, 24 at grade five and 16 at grade six.

It was not surprising that in-service education for a span of three, or even two grades required more time than for one grade. There were more topics to include and when the same topic appeared in more than one grade, more extensive treatment was needed. For teachers in a single system, increasing the time 50 to 70 percent makes possible coverage of the concepts in grades 4-6 at about the same depth as would be required for grade four alone. Multi-grade level groups tended to emphasize less the text presentation at any sigle grade level. While most teachers would profit from this greater investment in time, teachers are not favorable toward the additional burden of time, and they may resent the expenditure of time upon topics or aspects of topics not present in the curricula of the grade level they teach. These pressures favor in-service education for one grade exclusively when teachers are confronted with teaching a course such as SMSG *Mathematics for the Elementary School, Grade 4,* for the first time.

TIME FOR TEACHERS

Four precepts were derived from the 15 projects which apply to the mechanics of scheduling in-service study sessions. These are:

1. Conserve teachers' time.
2. Avoid conflict with other necessary activities.
3. Eliminate or postpone activities that have a lower priority.
4. Weigh carefully arbitrary decisions on scheduling caused by limitations in the time the resource person is available.

The following aspects of scheduling were present in the observed projects:

1. Monday and Thursday afternoons were used most often. One system held effective three-hour sessions on Saturday morning; no one used Friday or Sunday.

2. Where large groups are to meet in two or more sections, study the possibility for using this variation to achieve a better schedule "fit" for a larger number of teachers.
3. Collect adequate information on both school system schedule policies and all schedules to which the teachers are expected to conform prior to schedule making.
4. Get pertinent information including preference expressions from the teachers.

In several cases adjustments were made that freed the teacher from some teaching duties in order to schedule in-service sessions at a more convenient time; normally this was prior to pupil dismissal. Such arrangements were very difficult when several teachers came from the same school. When teachers were freed from classroom duties prior to dismissal of pupils, an earlier convening time enabled sessions to adjourn earlier or to last longer.

The length of sessions varied from as short as 1 hour to as long as 3 hours. Long sessions are difficult to achieve in combination with a full day of teaching. For teachers assigned to grades four, five or six, 2 hours seemed to be the maximum along with a full day of teaching. In general, sessions convened by 4:00 p.m. or earlier and none adjourned later than 6:00 p.m.

Though Saturday morning sessions are an encroachment on the teachers' private time, they avoid undue fatigue for teachers and long sessions are possible. If consideration is given to cost in time and effort for a teacher to reach an in-service session, those shorter than 90 minutes seem wasteful. The systems with greater experience in in-service education had the longer sessions.

The Length of In-Service Sessions For Each of Fifteen Projects

Length of Sessions (hours-minutes)	1-0	1-15	1-30	1-45	2-0	3-0
Number of Projects	5	2	4	0	3	1

Thirteen projects used a single meeting place. Convenience and routine were preferred over attempting to decrease or equalize the distance teachers would travel. Using a single location may decrease tardiness since travel factors become known.

ENOUGH TIME

The average total number of hours devoted to in-service sessions was 33.7. The observers see this as insufficient; 40 hours carefully planned and used would provide some attention to all topics in a single grade. These are approximations which must be interpreted with consideration for difference among teachers with respect to mathematics, and other variations. All agree that additional study of the mathematics involved would be beneficial.

In particular there is no contention that 40 hours of in-service education provides an adequate command of mathematics for the elementary teacher whose last formal study was a year of algebra and plane geometry in high school. By comparison a 3-unit college course would have about 50 class hours of instruction.

LEARN-TEACH MOTIVATION

The situation of simultaneous responsibility for both study and teaching was judged to provide strong motivation for teachers. As indicated earlier, "resource person" is a phrase used to identify those chosen to aid teachers in learning the unfamiliar mathematics encountered in these new programs. It is significant that resource persons in these 15 projects felt not only the immediate need of the teachers to understand mathematical concepts, but also their related need to be able to communicate these concepts to pupils. This recognition of a need to transmit mathematical understanding is very likely absent in many college mathematics classes, particularly where prospective teachers are a minority. Even in pre-service courses for teachers where months or years will elapse before the need to teach, the task of transmitting ideas to pupils will provide little motivation for the instructor and less for his students (prospective teachers). This immediacy of needs to understand and to teach in a restrictive time schedule served to force the attention of teachers and resource persons alike upon these essential matters.

THE RESOURCE PERSON

A total of 31 persons were involved in the 15 projects; 4 projects used one such person, 8 projects used two, one project used three and two projects used four each.

A principal resource person was identifiable for each project, hence the following table displays the vocation placement of resource persons.

Resource Persons Distributed by Full-Time Jobs

Full Time Job	Principal Resource Persons	Assisting Resource Persons
Mathematics Teacher Junior High School	1	1
Mathematics Teacher Senior High School	3	3
Mathematics Professor College	4	3
Mathematics Education Professor College	3	1
Mathematics Supervisor of County		2
General Supervisor of District	1	1
Principal Elementary School		3

In observers' opinions, the resource person was the most significant element in an in-service education program. His essential attributes were three in number: 1.) A knowledge of mathematics ample for his situation, 2.) The ability to communicate this knowledge to the teachers in his group, 3.) The ability to "join" the teachers in a scholastic partnership having common goals.

Concerning the first attribute the following indicators of adequacy appeared as useful: a.) Recent study of mathematics from the viewpoint of developing its structure — NSF institutes may have presented the opportunity to do so. b.) At least a baccalaureate degree in mathematics, c.) Recent involvement in work of some sustained and integral nature designed to improve the mathematics curriculum in a direction supported by modern authorities in school mathematics.

The second attribute is one which their prior students can attest to. The validity of these opinions is probably related to the similarity between the prior situation and the planned situation.

The third attribute is the least yielding to analysis, although much general analysis on this point does exist. Elementary teachers strive to be effective in several areas of instruction. This is one aspect of goals to be appreciated by resource persons — a value to be shared. Typically, elementary teachers see their pupils as personalities, not as mathematics students, and they see the learning situation through a wider spectrum of behavior than do teachers meeting pupils for one period for a single subject.

The observers saw it essential for the three attributes to exist simultaneously with the satisfactory resource person. Evidence of adequacy in knowledge of mathematics can be gotten more readily than can evidence on the second and third attributes. In fact the best judgment seems to require the corroboration of a trial. Fortunately, if the novitiate is successful in his first situation he is most likely to be successful with ensuing situations.

There was no intent or effort to rank the effectiveness of resource persons. Observers did not find a Ph. D in mathematics to be a handicap for a resource person, while lack of background was. Some successful resource persons had no degree beyond the BA or BS in mathematics. All successful resource persons had worked with mathematics that was compatible in nature with that being undertaken with their teacher groups; most frequently they had worked with similar material in junior or senior high school.

A LEARNING PROBLEM

The resource person is a central figure in a situation where teachers are being asked to reveal their status in mathematical understanding. Inference of unrealistic expectations is likely to inhibit teacher participation. One teacher group expressed the desire to exclude principals and other administrators from their in-service sessions; doing so promoted franker expression. All observers stressed the need for a secure situation for participating teachers.

Observers do not imply that resource persons should show an overly-solicitous concern for the teachers, but they must quickly estimate the teacher's mathematical knowledge. For only then are they likely to comprehend and deal helpfully with the wide variety of questions raised by teachers. The resource person needs the knowledge and skill to instruct teachers in the essential mathematics. But in doing so he must convey his desire for the teachers' success, and respect for the teacher's status.

WANTED: RESOURCE PERSONS

In nine projects the principal resource person was an employee of the school system and in the other six cases, he was employed for the in-service activity. In the nine cases using a member of the school staff, four used additional resource persons who were not regular employees of the school system.

School systems engaged in in-service education programs often wish to develop staff members to serve as resource persons. In doing so a valuable procedure was to team a potential resource person with one known to be capable in this role; this not only provided training but also a "test-run" for the prospective resource person without jeopardizing the success of the in-service venture.

The resource persons who were also staff members of the school system found it easier to do two things: 1.) visit the classrooms and work with the teacher and his class, 2.) meet with the teachers in small groups.

Observers reported that of the nine resource persons who were not college faculty members, six had studied additional mathematics through National Science Foundation opportunities. Two other instances of benefit came from industry-sponsored opportunities. Alert districts were encouraging potential resource persons to use the NSF created opportunities.

Among the very successful resource persons were those who had prior or concurrent experience teaching like mathematical concepts in junior senior high school. A uniquely valuable experience for some resource persons was writing the kind of material the in-service teachers were to teach.

The fact that successful in-service projects used two or more resource persons indicates that it is possible to sub-divide this job.

SOME EARMARKS

Generalizations about resource persons should be seen as directive rather than limiting. Their success is a function of a unique situation, and trial in the situation is the most revealing procedure to determine who is more capable of helping teachers.

The following generalizations seem to apply to most of the resource persons: They were committed to the idea that mathematics education could be improved. They had engaged in trial efforts to improve mathematics instruction. They were aware of different ventures toward improvement. They would experiment with different arrangements of content. They had knowledge and competency in mathematics related to the tasks they undertook. They were successful teachers of children and youth.

Finally, while the quality of the performance of a resource person was a function of the teachers involved, it seemed certain that a resource person who had demonstrated success in one situation was very likely to be successful in similar situations.

RECRUITING

How do teachers become involved in in-service education in mathematics? It may be their decision, it may be the decision of others. Observers agree that it is unwise to coerce teachers into teaching content they feel unprepared to handle. In the situations reported here, the content of the curriculum and the in-service education project were closely related. In this situation obtaining the teachers' consent to participate in teaching the new curriculum seemed best.

This consent was most readily achieved where there had been prior activities in in-service education in mathematics; where in-service education was a normal portion of operation in the school system; where the school system had improved its instructional program through use of wisely selected "new" programs not limited to mathematics; where the teaching staff were well informed concerning the nature of various efforts to improve the mathematics curriculum.

SOME OR ALL

Most frequently, administrators responsible for curriculum recommendations prefer to use pilot projects to evaluate and begin curriculum innovations. They wanted teachers who would "protect" instructional advancement of the pupil; who would contribute to the success of the project, and who would aid in the extension of this new phase of the curriculum if such extension is decided upon.

If the principle of consent is accepted, its simplest application is to call for volunteers. In general, this is not compatible with other objectives and the principle of consent may also be served when individuals are invited to participate; this was the general procedure when the selection was less than all of a group.

GROUP ENLISTMENT

Assignment of all teachers in a group to this new program appeared to be more successful when the teachers involved had good knowledge of the project. This was accomplished in a number of ways. For example, in two cases teachers had all participated in an in-service education program that increased their knowledge of appropriate mathematics and of the SMSG materials. In another project, classroom use of the new materials was delayed until the third school month while the teachers engaged in weekly preparation sessions. An important feature in this instance was postponement of entry of reluctant teachers into the program by scheduling exchanges of classes so that other teachers taught mathematics for them until they felt competent to take over. This was temporary use of some departmentalization. The reluctant teacher may also be a superior teacher.

Conditions were reported that were favorable to a category of teachers undertaking to teach the new curriculum together with a related program of in-service education. These conditions were not all from a single situation. The conditions are as follows:

1.) Recent in-service programs on the content of elementary school mathematics.
2.) Many teachers aware of the changing nature of the school mathematics curriculum.
3.) A history of the school system of instructional improvement from curricular innovation.
4.) School system arrangements to encourage in-service education — adjustments related to teachers' time and assignments, salary, conveniences and appropriateness of programs.
5.) Means to make adjustments for the "not ready" teacher — observing the principle of "consent".
6.) A school or district custom of regular periodic re-examination of major subject areas.

It would appear that several of these conditions are likely to exist in the more effective school systems since they apply to subjects other than mathematics.

WHY DO TEACHERS DO IT

What motivates teachers toward in-service education in mathematics? There is a tradition among teachers favorable to professional improvement; witness their enrollment in summer sessions at their own expense. This tradition has been stimulated by school policies effecting salary arrangements. Concerning arithmetic and mathematics, many teachers recognize this as an area for personal improvement; it has suffered neglect in pre-service training.

Teachers are aware of changes occurring at other grades and in other schools. They desire to have their pupils ready for what is ahead in subject matter and they have professional pride in a school system that compares favorably with others. In addition to this general kind of stimulation, it was a common practice among reporting systems to design their own in-service programs to obtain an agreement between the details of a teachers' job and professional preparation for it. A significant number of systems offer "salary credit points", that relate to periodic salary increases, to those who engage in the in-service education activities.

To summarize the reported factors stimulating teachers to undertake a new curriculum and in-service education related to it, the following aspects are briefly noted:

1. Approval of professional improvement for themselves and their colleagues.
2. Pride in the quality of education service given by their system.
3. Confidence in their professional leadership and a related willingness to respond to administrative requests.
4. A personal recognition of a need to strengthen their background in mathematics.
5. Knowledge of extensive changes occurring in the mathematics curriculum.
6. Desire to have their pupils enter other grades or schools well able to undertake the instruction given.
7. Desire to participate in a curriculum activity having merit and status.
8. Job status and salary are favorably influenced by participation.
9. Personal intellectual satisfaction.
10. Favor for study that fits closely the job being undertaken.
11. Social satisfaction of a joint venture with colleagues, and esteemed leadership.
12. An intrinsic interest in mathematics.
13. An intrinsic interest in the teaching-learning aspect of mathematics.

In none of the 15 projects were additional salary or bonus payments made to participating teachers while in every instance participation made a sizeable and definite demand on teachers' time.

Although an "average" teacher is impossible to define, it is probable that the teachers in the systems where indivduals were selected were above average. Only when all members of a category were chosen was there a normal cross section of teaching capability for that school system.

Concerning their age and experience in teaching there was great diversity in the whole group; teaching experience varied from zero to thirty-seven years, with a mean of seven years. Nearly all of the teachers had baccalaureate degrees and some had master's degrees, but few had special training in mathematics.

BETTER WORKING CONDITIONS

Observer responses to a request for suggestions of changes that the teachers would approve cover five aspects. These were: 1.) expansion of information about and orientation to changes in mathematics instruction; 2.) reduction of demands on teachers' time and more time for in-service education; 3.) conservation of teacher effort through sharing of special exercises prepared and the like; 4.) helping each teacher with his self-appraisal, and 5.) additional rewards in salary now or in the immediate future.

WHAT AND HOW

The in-service education sessions with the resource persons had primary responsibility for an orderly development of mathematical ideas, but the subjects considered fell into three categories. These were:

1. The specific mathematics to be taught to pupils.
2. The process of instruction incorporating this content.
3. Analysis of the instructional experiences encountered by the teachers.

The mathematical concepts treated were those to be taught, hence for teachers using SMSG *Mathematics for the Elementary School, Grade 4*, the ten units which it includes were the outline to be covered and the *Teachers Commentary* was the important resource material.

For groups that included teachers at grade five and six, additional topics were included and those begun at grade four were expanded.

As to emphasis given in teacher resource-person sessions, first place was given to development of mathematical concepts and second place to treatment of these concepts in the process of teaching, while a third and smaller group thought they had tied together these matters so integration had been accomplished.

One project spanning grade 4, 5 and 6 used SMSG "Studies in Mathematics", *Number Systems* as a teachers' textbook with geometric topics coming from the Commentaries.

"Homework" for teachers had contrasting receptions, one as a necessity, the other as an imposition. A tentative approach to homework is recommended; apply it with great sensitivity to the effective use of the teacher's time.

In several projects the preparation of test items was recommended because it focused teacher attention on essential matters as well as provided means for evaluation.

In addition to extensive use of chalkboard and duplicated materials, other instructional aids were used with teachers. These included geometric models, abaci, a finite mathematical system using materials like those described in Kelley's *Modern Algebra,* television instruction, films, charts, tapes and transparencies for overhead projectors.

NOTE ON SMSG TESTS

In some of the studies reported in this Newsletter and also Newsletter No. 10, mention is made of special tests devised by SMSG. These will eventually all be published. At present, however, these tests must be kept secure and cannot be released. There are a number of reasons for this. First, and most impo tant, a number of items from these tests will be incorporated in special tests being developed for use in the National Longitudinal Study of Mathematical Abilities, described in Newsletter No. 11.

In the second place, no norms for these tests are available so that scores on these tests cannot be interpreted. Also, in many cases the tests are devoted only to those special aspects in which SMSG courses differ from conventional ones.

It is hoped that one outcome of the National Longitudinal Study of Mathematical Abilities will be specifications for and samples of tests which will be more useful than those constructed so far.

CORRESPONDENCE COURSES

In 1959-60 and again in 1960-61, the Minnesota National Laboratory carried out for SMSG a study of the feasability of providing corresponding courses in ninth grade algebra and tenth grade geometry, using the SMSG texts for these grades. The purpose of the study was to determine whether high ability students could master these courses by correspondence.

The results were quite satisfactory, and these courses are now available, not only from the University of Minnesota but also from the University of Wisconsin, which cooperated in the preparation of the study guides for the courses.

In addition, correspondence courses for teachers of algebra and geometry are being prepared by the University of Wisconsin.

For further information, write:

Correspondence Study Department
University of Minnesota
Minneapolis 14, Minnesota

or

Extension Mathematics Department
University of Wisconsin Extension Division
Madison 6, Wisconsin

If you are not now on the SMSG mailing list but wish to receive further issues of the SMSG Newsletter, please request, by means of a postal card, that your name be added to the mailing list. Address request to:

SMSG Cedar Hall
Stanford University
Stanford, California

SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 19

September 1964

REPORT OF A SURVEY OF IN-SERVICE PROGRAMS FOR MATHEMATICS TEACHERS

Financial Support of the School Mathematics Study Group has been provided by the National Science Foundation

CONTENTS

Since its inception the School Mathematics Study Group has considered one of its major concerns to be the problem of providing inservice teachers with the understanding of mathematics needed to teach an up-to-date curriculum. Two extracts from earlier SMSG Newsletters express the SMSG point of view on this matter.

The first extract is taken from Newsletter No. 5, November 1960, which was devoted to a report on in-service experiences in the first SMSG Experimental Centers.

> "However, most of these teachers face a common problem when teaching SMSG texts, or any other improved curriculum materials, for the first time. This is the problem of obtaining the additional preparation in mathematics which these texts require a teacher to possess. Most teachers, through no fault of their own, were not provided in their pre-service preparation with this kind of mathematics. Consequently, most teachers need some assistance either before or during their first use of these texts.
>
> "Some teachers will be able to obtain this additional preparation in summer school courses or in Institutes such as those sponsored by the National Science Foundation. However, most teachers will have to obtain this assistance through in-service programs arranged by the school or school system in which the new materials are to be taughr."

The second extract is taken from Newsletter No. 18, April 1964, which contained a report on use of the SMSG materials for grades K-3.

> "In-service classes pertaining to the mathemarical content of the K-3 program have been held for the participating teachers in the project centers. These have been indispensable. It simply is not possible to implement a program of the kind reflected in Mathematics for the Elementary School, K-3, without undergirding it with a substantial program of in-service training for the teachers involved."

Cases are known where schools have instituted new mathematics programs without due regard for the point of view expressed above. In many of these cases the results were unfortunate for both the teachers and the students.

However, most schools are well aware of the importance of in-service programs for mathematics teachers in any attempt to improve mathematics programs for students. A large variety of in-service programs has been instituted in recent years.

It seemed to the SMSG Panel on Teacher Training Materials that a survey of these programs might well reveal ideas and practices deserving wide notice.

This issue of the SMSG Newsletter is devoted to a report of this survey.

During the past summer a small team of writers prepared a preliminary version of a sample in-service course for 7th grade teachers which incorporates many of the ideas revealed by this survey. Further information about this course, which is now being pilot tested, will appear in a later issue of this Newsletter.

REPORT OF SURVEY COMMITTEE OF IN-SERVICE MATHEMATICS PROGRAMS OF THE PANEL ON TEACHER TRAINING MATERIALS OF SMSG

The work of this committee (Nick L. Massey, Seattle Public Schools, Irene St. Clair, Texas Education Agency, Max Sobel, Montclair State College, and John Wagner, Michigan State University) was activated by a charge from the Panel on Teacher Training Materials of SMSG:

> A more careful and more extensive survey should be made of in-service programs now in use. This should include not only in-service programs for classroom teachers but also programs designed to train individuals to staff in-service programs. These will be surveyed by the Panel and the most promising of them will be listed and described in a publication which will be given wide circulation, perhaps an SMSG Newsletter.

Initially the committee decided to contact each state supervisor and other selected personnel to secure the names of people responsible for successful in-service programs. Information was also solicited by announcements in *The Arithmetic Teacher, The Mathematics Teacher,* and *School Science and Mathematics.* Particular attention was paid to locally sponsored programs and less attention to N.S.F. institute activities. The latter are well documented in the N.S.F. report files and tend to be of a rather straightforward "text-lecture" subject matter class nature. It was decided to document those in-service training programs which were sponsored by state departments. A survey was also instituted to find all information possible about those programs designed to train individuals to staff in-service programs.

Attention was paid to documented reports on in-service activities such as "A Report on In-Service Education" in SMSG Newsletter No. 15, *In-Service Education of High School Mathematics Teachers* (a report of the U.S. Department of HEW of a conference, March 17-19, 1960), "In-service Education of Mathematics Teachers" by Veryl Schult and Theodore L. Abell, *School Life,* July 1963, and a forthcoming formal report of a conference, March 7-8, 1963, alluded to in the Schult-Abell article. It was felt that no progress would be achieved in retracing the same ground covered by these other investigations. On the other hand, these other surveys might well indicate ground that was unexplored.

A questionnaire was drafted which embodied the questions we would pursue with respect to locally sponsored programs. This questionnaire is shown later (p. 25, 26, 27). The reports of the state sponsored programs are categorized and the reports of the five institutions designed to train leaders are presented in a summary fashion.

The questionnaire was sent to those places which had been indicated by "authorities" as having successful programs. The percentage of returns was extremely surprising to the several of us familiar with questionnaire type surveys—the order of magnitude of 80 per cent! This return, the extemporaneous remarks of the respondents, plus their voluminous attachment of materials and their comments, the manifest interest of state supervisors for the report would all seem to indicate the timeliness of this study. It was decided at the meeting to stratify the responses under the three main teacher levels—Elementary, Junior High, and Senior High—i.e., questions involving length of course, association of course with adoption of new texts, assignment of homework, availability of credit, etc.

The main body of the report is as follows: a report of the survey of the state department sponsored programs and comments concerning these reports as they relate to the work of SMSG, a report of programs designed to train leaders of in-service programs, plus comments, and the same for locally sponsored programs.

MATHEMATICS IN-SERVICE PRACTICES AT STATE LEVEL IN SOME STATES

Types of In-Service Education and Materials Used

A. Alabama

- *1. Alabama conducts on-campus weekly meetings through the mathematics department of the University of Alabama and Jacksonville State College. Funds are provided by NSF.
- **2. Textbook materials for the Alabama in-service programs are SMSG materials, Volumes 1 and 2 of Junior High School Mathematics and Algebra.

B. Alaska

1. Alaska is introducing in-service training by means of new elementary curriculum guides which were produced in a workshop.
2. No materials mentioned.

C. Arkansas

1. Arkansas holds in-service classes two hours per week for six weeks, with about 40 teachers in each class.
2. SMSG materials are being used, but have been revised into the format of 18 programmed homework lessons by a staff member of the University of Virginia.

D. California

1. Through the State Department of Education, California, in cooperation with the University of California, operates in-service training programs composed of 28 one-half hour lessons which have been taped and filmed. The series is presently being broadcast over 13 television stations in California. A second series of television in-service programs was prepared by Dr. Stewart Moredock using SMSG materials.
2. The California State Department and the University of California have cooperatively produced a series called "Understanding Rational Numbers". In addition materials have been produced from the SMSG studies.

* Types of In-service Education
** Materials Used

E. Delaware

1. Six in-service programs have been established in Delaware under the teacher education and Professional Services Division of the State Department, such courses qualify participants for credit on the professional improvement scale. Other programs were organized on a county basis or on a district basis; some of these were at the elementary level and others on the secondary level.
2. Materials used in Delaware were the SMSG Algebra and 7th grade materials and new commercial text materials.

F. Idaho

1. College courses and Project Idaho courses. In Project Idaho, classes are conducted at 23 centers sponsored by local school districts and designed for elementary teachers.
2. Course materials were prepared by a committee and instructors were brought into a central meeting place for orientation before the classes began.

G. Illinois

1. Courses operated in 22 centers and taught by staff members of the State Department of Education and college instructors.
2. Number and Operation Handbook and library books provided for duration.

H. Iowa

1. Iowa has organized regional in-service college workshops to which specially screened teachers have been invited. Purposes are to become acquainted with the new concepts and to develop new techniques for teaching. These workshops are developed in phases: Phase I, Phase II, and Phase III.
2. The State Committee in cooperation with the instructors are selecting the content.

I. Minnesota

1. In-service in Minnesota is operated chiefly by means of correspondence courses.
2. Programed correspondence courses have been prepared in aigebra and in geometry. These courses are currently being piloted by 80 teachers in a five-state area. Following the pilot test, the courses are to be revised and distributed widely. The project is financed by NSF.

J. Mississippi

1. In-service programs offered in Mississippi are college oriented and follow the category of regional courses taught on campus or by extension at off-campus centers.
2. Materials used in all secondary courses are the University of Maryland's Mathematics Program materials.

K. Missouri

1. Missouri conducts in-service programs for a duration of 10 weeks offered by the mathematics consultants in selected geographical areas. During the past school year approximately 550 teachers from 85 school districts participated.
2. Apparently, the Missouri consultants prepare the material, using the lecture method; outline of scope and sequence, hand-out materials and texts as they proceed.

L. Montana

1. Montana has been conducting one-day workshops in elementary mathematics and on-campus workshops for teachers at the universities.
2. Various materials are used with the predominance of SMSG materials.

M. Nevada

1. Nevada has sponsored for two years in-service training programs located regionally and directed to the elementary level. Programs are successful to the degree that teachers are convinced that the new programs are an improvement; attitudes of teachers changed; they feel secure in handling the new programs. It is important that one or two units of modern mathematics be tried by the teachers in their classrooms as in a laboratory situation.
2. The course content has been developed by the mathematics consultant; it follows closely the SMSG Studies in Mathematics, Volume 6. In addition current commercial textbooks and enrichment materials are used to supplement the lectures.

N. New Mexico

1. Most in-service programs in New Mexico are operated as extension courses for college credit.
2. Instructional materials have been the SMSG publication for teachers.

O. North Carolina

1. North Carolina conducts in-service programs on college campuses and in most cases, provides for college credit. Programed materials were used during the spring of 1963.
2. Programed materials that were used at the elementary and secondary workshops were published by: Teaching Materials Corporation; Science Research Associates; Encyclopedia Britannica Films; Holt, Rinehart and Winston; McGraw-Hill Book Company.

P. Ohio

1. Ohio provides financial assistance for in-service programs in mathematics sponsored by a city or county school system. The instructor-consultant is usually a college instructor and determines the course content according to the needs and desires of the local situation.
2. Reports indicate that stressing the ideas of modern mathematics, introducing material that could be used in the classroom, and providing bibliographies served as an immediate help to teachers.

Q. Oregon

1. In-service programs in Oregon take on a variety of formats; ETV; small groups of teachers on individual campuses, and extension classes conducted through the State System of Higher Education.
2. Materials are varied, usually being selected by the instructor to meet the local needs.

R. Tennessee

1. In Tennessee in-service programs have become a function carried out by the local school systems. A series of one-day workshops provided orientation for such programs.
2. Materials used in the workshop include a Scrambled Book dealing with such subjects as set, number, and numeration systems. Evaluation forms of a test covering these topics were given before and after to the participants in workshops.

S. Texas

1. In-service education in Texas is provided according to several plans: Locally-sponsored courses; college courses, both on campus and extension; regionally-sponsored courses; and courses offered by the Texas Education Agency at centers throughout the state. In addition a self-instructional program for elementary teachers has been in operation by the means of supplying individual workbooks to elementary teachers throughout the state.
2. Courses sponsored by the Texas Education Agency use SMSG materials at the proper grade level. For the elementary course, materials were purchased from a commercial company with the intent that they were self-instructional and that they provide worksheets for pupils.

T. Virginia

1. In Virginia in-service training programs are provided for elementary teachers in the area of modern mathematical use of preparation of multi-sensory equipment in the classroom; junior high and senior high school mathematics teachers.
2. Materials provided by Virginia for the elementary teachers were SMSG Studies in Mathematics, Volume 9. For the second one regarding multi-sensory equipment, there is no textbook provided. For the secondary school teachers materials are chosen to be compatible with needs of the local system involved.

U. Washington

1. Washington holds one-day regional work conferences with elementary teachers and administrators to implement future in-service programs that will make use of the SMSG films.
2. Washington uses SMSG Studies in Mathematics, Volume 9 and the series of 30 half-hour films.

V. West Virginia

1. West Virginia reports a program for teachers of grades 1-8; administrators are also included.

No college credit is given, but a Certificate of Merit is presented to participants. Meetings were held regionally according to demand. There were 10 two-hour class periods.

2. West Virginia uses lesson booklets containing the content material prepared by the staff at the State Department. A method library is made available to participants during the operation of the program.

W. Wisconsin

1. Television courses and college courses sponsored by the University of Wisconsin.
2. Special courses, Patterns in Arithmetic, consisting of more than 60 programs on two different levels of elementary instruction have been prepared and devised for the in-service program.

COMMENTS ON PROGRAMS AT STATE LEVELS

It is not surprising that a goodly number of the state department sponsored activities are oriented toward preparing teac · rs to handle newly adopted materials. It may be well that in-service activities are motivated by the contemporary demand of the classroom situation. A predominance of SMSG "teacher-training" materials are in use, i.e., *Studies in Mathematics*. These materials were designed to accommodate the classroom teacher in handling specific courses, e.g., "Intermediate Algebra" and "Geometry". State sponsored programs could profit from more materials of the *Studies in Mathematics* nature perhaps supplemented by convenient means of handling larger groups (area wide workshops) such as prepared overlays for projectors. In many cases these state sponsored courses are of short duration and quick means should be available to expedite the work.

In some cases television has been tried. Some exploration should be attempted as to the format and/or type of materials best adapted to learning mathematics via TV. It may be that correspondence courses (i.e. Minnesota and Wisconsin) could be run tandem with TV. (Any difference with slides and overlays with regular lecture as contrasted with their use with TV?)

The upshot is that state department sponsored courses look to preparing the teacher in the rather early future with "newer" materials. The slight indication here is that the materials are a little more "snappy" than the longer route suggested by more formal text materials and are oriented toward the materials actually in use in the classroom.

TRAINING OF LEADERS FOR ELEMENTARY IN-SERVICE INSTITUTES

A promising practice that has developed in several sections of the country is the training of secondary school teachers as leaders of in-service programs for elementary teachers. Following are descriptions of several such programs.

1. San Jose State College, San Jose, California

San Jose is in the midst of a second Academic Year Institute, supported by NSF, designed to prepare selected junior high school teachers to present in-service training for elementary school teachers.

The program provides for 20 teachers and 3 supervisors who have had a minimum of 3 years experience in grades 7 to 9, as well as maturity in undergraduate mathematics equivalent to 10 semester units of analytic geometry and calculus. Applicants are required to supply information concerning the potential demand for their services as leaders of in-service programs. At least one elementary school district must indicate tentative plans for such service.

The training program consists of a four week preliminary refresher course for those who have not studied mathematics in recent years. Thereafter the core of courses includes:

- Basic Ideas of Elementary Mathematics (Grades K-8)
- Foundations of Mathematics
- Geometry
- Introduction to Modern Algebra
- Problem Solving: Theory and Practice
- Basic Concepts of Mathematics for Senior High School (Grades 9-12)
- Observation and Practice in Teaching of Contemporary Mathematics Programs

The latter course attempts to give the students an opportunity to discuss and develop an in-service course, observe a class in mathematics for pre-service elementary school teachers, examine published materials of such groups as SMSG, UICSM, etc., and meet in a weekly seminar to discuss issues relevant to the above. The SMSG films together with SM IX are discussed.

For further information, write to:
Dr. Max Kramer
Department of Mathematics
San Jose State College
San Jose, California

2. Washington State University, Pullman, Washington

For the past three summers a Seminar on the Foundations of Arithmetic has been conducted as a special aspect of the Summer NSF Institutes for high school mathematics teachers. Participants selected for this special seminar had to qualify on the basis of:

a) mathematical ability and knowledge;

b) enthusiastic desire to conduct courses for elementary school teachers; and

c) explicit written understandings from their superintendents that they and their elementary school colleagues will be given the time, opportunity, and local financial support to conduct such programs during the following school year.

The seminar was based on the text ***Fundamental Concepts of Arithmetic*** by Hacker, Barnes, and Long. (Prentice-Hall, 1963)

To date, indications are that the participants who have participated in the training seminars have reacted very favorably, and that the elementary teachers trained to date have been enthusiastically in favor of the program. As a result of these Institutes, over 500 elementary school teachers have been involved in local in-service programs during the last three academic years.

For further information, write to:
Professor W. E. Barnes
Department of Mathematics
Washington State University
Pullman, Washington

3. University of Vermont

In the summer of 1963, a group of secondary school teachers participated in a NSF Institute at the University of Vermont. These teachers in turn conducted local in-service courses for elementary school teachers and were visited by members of the University staff.

Thirty-one participants attended the summer program, and twenty-five of these conducted substantial in-service work with teachers and administrators in their local districts. The total number of teachers involved in the in-service programs was 952. The number of meetings for these courses ranged from 7 to 18; the number of hours each in-service group met ranged from 10 to 46.

The major objective of these programs was to present the subject matter of mathematics which the Committee on the Undergraduate Program in Mathematics (CUPM) and other groups regard as essential for elementary school teachers to have in order for them to successfully teach current elementary school programs. In all of the in-service work the problems of how to use the knowledge gained to improve mathematics programs for children was considered, but for the most part the emphasis was on mathematics subject matter.

For further information, write to:
Dr. Ruth K. Izzo
Department of Mathematics
University of Vermont
Burlington, Vermont

4. University of Washington, Seattle, Washington

To help achieve in a relatively short time the goal of having available high school mathematics teachers prepared to work with elementary teachers, a series of three day workshops was held. The program for the three days included the following:

a) a discussion of the need for reform of elementary school mathematics;
b) an examination of the general philosophies of current curriculum reforms;
c) a grade-by-grade survey of current elementary school mathematics as compared with traditional programs;
d) an examination of elementary text materials; and
e) a discussion of problems relating to administration of such programs.

These institutes were held in the spring of 1963, were supported by NSF, and drew participants from school districts within the state.

For further information, write to:
Professor Roy Dubisch
Professor of Mathematics
University of Washington
Seattle, Washington

5. The College of Idaho, Caldwell, Idaho

"Project Idaho", financed by a grant from NCTM, brought 23 of Idaho's more qualified secondary school teachers of mathematics to the campus of the College of Idaho for an intensive two-week workshop in June, 1963. These teachers then returned to their home districts and during the academic year 1963-64 conducted in-service classes for elementary teachers.

The two-week workshop consisted of morning sessions on subject matter content with an emphasis on the structure of the number system. Afternoon sessions dealt with discussions of available materials, methods of organizing workshops, and elementary school curricula.

For further information, write to:
Dr. Boyd Henry
Department of Mathematics
College of Idaho
Caldwell, Idaho

COMMENTS ON LEADER TRAINING

Preparing competent high school teachers as leaders of in-service programs for teachers of elementary school mathematics appears to be a promising trend. The programs described here ranged from three day training programs to an academic year institute. Obviously no general pattern exists and further experimentation is desirable. Nevertheless, this appears to be a worthwhile means of reaching the numerous teachers of elementary mathematics in the country.

LOCALLY SPONSORED PROGRAMS

Responses ranged geographically from New York to California and Texas to Wisconsin, and included large and small school districts, college and university people (both mathematics and education departments), supervisors and teachers.

In attempting to summarize responses, the first efforts were made to classify the results by three levels: elementary, junior high, and senior high. This, however, proved difficult since most in-service efforts included teachers from more than one of these levels. Six classifications resulted, and the other information has been presented under these headings. They are elementary only, elementary and junior high, junior high only, junior and senior high, elementary-junior high-senior high, and senior high only.

Of all the questionnaires returned, 91 were included in the tally, others being eliminated for various reasons.

Most courses, 37%, were offered for elementary teachers only, followed by the elementary and junior high combination, 29%; elementary-junior high-senior high, 14%; and junior-senior high, 11%. The majority, 62%, were classified as school district course; the remaining 38% were college courses. Of the courses for elementary teachers only, 79% were school district offerings which for the elementary-junior combination 54% were school district, and 85% of elementary-junior-senior were school district. Colleges seemed to provide more for the junior and senior high school teachers and less for the elementary teachers. They also offered few of the broad "all-category" courses. Only 18% of the total were compulsory in nature.

Statistics on the size of classes seemed not too significant, however 62% of the broad elementary-junior-senior combination were large, 100 or more. The elementary only were either small (1-49), 38%; or large (100 or more), 35%. Totally, 48% were small (1-49), 21% were in the 50-99 size, and 31% were 100 or more.

Class length in hours was interesting in that 59% were 25 hours or more, and 35% were 11-24 hours. Only 6% were 10 hours or less.

In the combination class (more than one level), 73% were separated for at least part of the instruction.

The major part of this results from the elementary-junior combination were 88% of the class were separated part of the time.

About half, 53%, of the total courses were concerned with an adoption and not all these directly.

Most instructors, 82%, were either from colleges, (56%) or were in some school supervisory position, (26%). 18% of the instructors were school district teachers.

Almost all said that homework was assigned. Totally, 81% responded yes to this question.

Most courses, 66%, were offered for credit. Of these, 65% were for college credit (about 50-50 graduate vs undergraduate) and 35% offered school district credit only.

53% of the courses totally (65% of the elementary) were not oriented toward any specific program; 40% were oriented toward some SMSG course, leaving only 8% oriented toward any commercial program.

When it came to use of a textbook, 69% said they used some text.

With regard to textbooks, only a few were mentioned more than once. SMSG publications were cited more than any other single group of books. Although some of the SMSG student texts were used in courses the various Studies in Mathematics were referred to most often.

The following is a listing of books mentioned more than once with the frequency of mention in parentheses.

SMSG—SM IX, *Brief Course for Elementary Teachers* .... (12)

Peterson and Hashisaki, *Theory of Arithmetic* (9)

Brumfiel, et. al. *Fundamental Concepts of Elementary Mathematics* .... (5)

SMSG—SM VII, *Intuitive Geometry* .... (5)

SM VI, *Number Systems* .... (5)

SM VIII, *Concepts of Algebra* .... (2)

SM V, *Concepts of Informal Geometry* .... (2)

SM IV, *Geometry* .... (2)

SM III, *Structure of Elementary Algebra* .... (2)

Webber and Brown, *Basic Concepts of Mathematics* .... (5)

SMSG, Elementary texts grades 4, 5, 6 .... (4)

Meserve and Sobel, *Modern Mathematics for Secondary Teachers* .... (4)

SMSG, *Geometry* .... (4)

Hacker, et. al., *Fundamental Concepts of Arithmetic* .... (2)

Banks, *Teaching and Learning Arithmetic* .... (2)

Van Engen and Gibb, *Charting the Course for Arithmetic* .... (2)

SRA materials .... (2)

SMSG, *First Course in Algebra* .... (2)

SMSG, *Mathematics for Junior High School*, V. I, II .... (2)

McCoy, *Introduction to Modern Algebra* .... (2)

Goldberg, *Probability* .... (2)

The responses to the question regarding topics taught, while not different in overall content, differed enough in detail to defy classification. In general, most courses for the elementary level covered the usual topics of numeration, operations and their properties, the number system, and an introduction to a few of the concepts of algebra and geometry. Courses for the secondary level usually were more restricted in content, being designed around a particular course, i.e., Elementary Algebra, Geometry, etc.

Several respondents mentioned the need for books written specifically for certain grade levels, showing a dissatisfaction with most texts currently available for in-service work. Films and overlays keyed to a book were mentioned several times. A few people would like to see some films of actual lessons being taught.

The apparent lack of more than a few suitable textbooks and other instructional materials was clear from the fact that approximately 73% of the responses to the request to name useful commercial materials listed nothing.

Tabulation of Responses from 91 Questionnaires Classified into Six Grade Level Groups

	Elementary Only	Elementary-Junior High Only	Junior High Only	Junior and Senior High Only	Elementary-Junior High-Senior High Combination	Senior High Only	TOTAL
Level of Course	34	26	4	10	13	4	91
Type of Course							
College	7	14	3	6	2	3	35
Sch. Dist.	27	12	6	4	11	1	56
Compulsory	6	5	1	2	2	0	16
Size							
1-49	13	16	2	9	2	2	44
50-99	9	5	0	1	3	1	19
100 up	12	5	2	0	8	1	28
Length							
0-10	2	2	0	0	1	0	5
11-24	13	7	1	3	8	0	32
25 up	19	17	3	7	4	4	54
Levels Separated?							
Yes	—	3	—	3	7	—	13
No	—	23	—	7	6	—	36
Adoption							
Yes	20	15	1	5	6	1	48
No	14	11	3	5	7	3	43
Instructor							
College	13	16	3	9	7	3	51
School Sup.	16	2	1	0	5	0	24
Teacher	5	8	0	1	1	1	16
Homework?							
Yes	29	20	4	9	9	3	74
No	5	6	0	1	4	1	17
Credit?							
No	16	6	0	1	8	0	31
Yes	18	20	4	9	5	4	60
College	8	17	3	5	3	3	39
Sch. Dist.	10	3	1	4	2	1	21
Program							
None	22	18	0	1	5	2	48
SMSG	8	7	4	8	7	2	36
Commercial	4	1	0	1	1	0	7
Textbook?							
Yes	23	18	4	10	7	1	63
No	11	8	0	0	6	3	28

SCHOOL MATHEMATICS STUDY GROUP

In-Service Committee Survey

Questionnaire

1. Course title (if any) ____________
2. Primary emphasis (if not obvious from title) ____________
3. Type of course or program:
   college____ school district in-service____
   other ____________
4. Voluntary____ or Compulsory____
5. Approximate number of teachers involved ____
6. Length of course:
   Number of sessions ____________
   Length of each session ____________
   Frequency of sessions ____________
7. Released time for teachers:
   None____ Total____
   Partial____ How much?____
8. Level of teachers:
   Elementary____ Junior high____ Senior high____
   Were different levels separated? Yes____ No____
9. How was course financed?
   NDEA____ State____ Local____
   Student fees____
   Other ____________
10. Was course evaluated?
    Yes____ No____
    by instructor?____
    teachers?____
    other?____
    how? ____________
11. Was this course associated with adoption of new text(s)?
    Yes____ No____
    forthcoming____
    completed____
    when?____
12. If this course will be repeated, what plans (if any) are there for modification?
    ____

13. Source of instructor:
college_____ school supervisor_____
elementary teacher_____ junior high teacher_____
senior high teacher_____ jr. college teacher_____
other _____

14. Was homework assigned?
YES_____ NO_____
how much (in time)?_____
how often?_____
collected?_____

15. Was credit available?
YES_____ NO_____
college? Yes_____ No_____
undergraduate_____ graduate_____
school district? Yes_____ No_____
acceptable on salary schedule?
Yes_____ No_____

16. Was course oriented toward any specific program?
None_____ SMSG_____ UICSM_____
University of Maryland_____ Ball State_____
Commercial_____
which one? _____
Other _____

17. Did you use a textbook?
A. YES_____
which book? _____
how much of book (half, two-thirds, special chapters)? _____
_____
was it adequate? Yes_____ No_____
B. NO_____
what materials *were* used (if any)?_____
_____
C. If possible, list main topics and approximate time spent on each.
_____
_____
_____
_____
_____

18. What would you like to see available for in-service work? Books? (content, degree of rigor, etc.), films?, filmstrips?, overlays?, etc. (Use back of sheet if necessary.)
_____ _____
_____ _____
_____ _____
_____ _____

19. Any commercial material other than what ma[y] have been mentioned in question 17 which seem suitable?
_____
_____
_____

20. Would you like to receive a copy of any report prepared from this information?
Yes_____ No_____
Name _____
Title _____
Address _____
_____(zip code)_____

NOTE: A copy of any available printed materials would be greatly appreciated.

COMMENTS ON LOCALLY SPONSORED PROGRAMS

Certainly the on-going pattern of in-service education activities seems to be moving in the direction of stratified grouping—elementary teachers as contrasted with junior high—and less on omnibus courses across wide areas of grade levels. The central motivation in many of these endeavors also appears to be toward the adoption of new classroom materials. Even though no particular program may appear to dominate, it is interesting that the adoption of newer materials in many cases serves to coalesce faculties into such study activities.

Adequate time appears to exist to allow the development of in-service courses (59% were of 25 hours duration or more) but with a not too surprising number of the instructors coming from the corps of contemporary teachers in public schools. Instructors apparently are available only from the colleges or high supervisory echelons — again the need of a cadre of available instructors!

Interestingly enough the assignment of homework is incident to many of the existing courses—this favorably bears out the feeling of many who feel that mathematics is not and should not be a passive activity for student and/or teacher.

SMSG "Studies in Mathematics"—type books seem to dominate the selection of materials for these activities. It should be reminded that these are books explicitly designed for "in-service" work. A great deal of the existing commercially available literature seems not ideally designed for the peculiar requirements for the in-service education of the on-the-job teacher (73% of the respondents failed to list any *adequate* commercial materials). The responses would seem to indicate some need of more directly oriented in-service "packages." Package?—books, pamphlets, illustration of existing classroom courses of study, intimations of pedagogy, supplementary material of any sort which will expedite the teacher to the job at hand. A combination of reported larger classes as superimposed on a small supply of instructors would indicate that these "packages" should lend themselves to large groups (Some in-service classes report attendance of 60-80 students attending regularly for 10 or more sessions).

This brings up another critical point. Too little time, resources, or efforts seem to be concentrated on the problem of discovering, recruiting, and training talent for leading in-service institutes (this seems peculiarly acute at the elementary level). From this study there would appear to be plenty of room for colleges or larger school districts to accept the job of accepting the task of educating personnel for the job of staffing in-service education operations. In addition the intensive and far-flung activities of the various state departments of education would indicate a fruitful liaison between colleges and school districts which could be enlisted for the development and expansion at all levels of in-service education of teachers, i.e., recruitment of leadership as well as efficient design of organization.

SCHOOL MATHEMATICS STUDY GROUP

Newsletter No. 30

March 1969

STATUS REPORTS

RECENT PUBLICATIONS

CONTENTS

STATUS REPORTS

It has been over two years since a report was made to the mathematical community (SMSG Newsletter No. 24) on the status of various SMSG projects. A good deal was accomplished during those two years and it is appropriate to once again offer a general progress report.

Three major activities are now being carried on. One of these is a four year longitudinal study of mathematics learning in the primary grades; the second is the analysis of the enormous amount of data collected in the National Longitudinal Study of Mathematical Abilities; the third is the preparation of a new secondary school program in which the sequence is determined not by the customary grade placement of topics but rather by the wish to provide a better integration of mathematical ideas. A separate status report on each of these projects appears below.

No less important, although on a smaller scale, are the other current SMSG activities. Volume 20 of the Monograph Series has been published and 4 other monographs are being prepared for publication. A new series of translations of Russian articles on the psychology of mathematics learning is described more fully below.

Research on mathematics education is receiving increased emphasis. This is evidenced in part by the Journal of Abstracts described below. A number of small research studies are now being carried out at SMSG headquarters and the volume and size of these studies will undoubtedly increase in the next few years. Reports on these will be announced in future SMSG Newsletters.

PRIMARY SCHOOL LONGITUDINAL STUDY PROGRESS REPORT

I. *Introduction*

In the fall of 1966, the School Mathematics Study Group embarked upon a four-year longitudinal study of mathematical learning in the primary grades. The primary purpose of this study is to assess children's progress in learning particular mathematical ideas during the beginning school years.

This interim report includes an overview of the design, current work, and that planned for the remainder of the study. The results presented are those from the first year of the study, the kindergarten year, only.

II. *Description of the Sample and Research Design*

The study population includes approximately 2000 children entering kindergarten in selected schools of two large cities. The schools selected met two criteria: they drew on residential areas which could be described as either lower or middle income groups, and each particular group of elementary schools fed into a common junior high school. One lower and one middle income group in each city are using the School Mathematics Study Group curriculum, and the other, comparable groups are using the Science Research Associates program (Greater Cleveland Mathematics).

The following diagram shows the groupings of children in the study:

	Curriculum	
Income Level	SMSG Lower	SRA Lower
	SMSG Middle	SRA Middle

Thus, we will be looking at curriculum and socioeconomic level of the groups as they affect mathematics performance. In addition, intelligence measures and later in the study, attitude measures will be related to mathematics achievement on standardized tests as well as on tests developed during the pilot years, 1964–66, and throughout the course of this longitudinal study.

The measurement of teacher variables is limited to their use of and adherence to the curriculum they are teaching. These data are being obtained both through questionnaires and some classroom observation. Control of the variable of teacher preparation for teaching modern elementary mathematics was exercised by the provision of inservice training to all teachers within the study who had not had this training within the previous two to three years. The inservice training was provided by teams of two instructors, a mathematician and an experienced elementary teacher with considerable knowledge of "modern" mathematics, with the former presenting the mathematical ideas and the latter relating these ideas to pedagogy at the kindergarten and first grade levels. Separate inservice courses were scheduled for the teachers using the two different curricula so that applications of the mathematics background could be made directly to the curriculum being used by each teacher.

The following diagram shows the variables being studied, with the independent or predictor variables listed first, and the dependent or criterion variables listed below.

Independent Variables

Student
- Past performance } Readiness
- Cognitive development } Readiness
- Intellectual performance
- Behavior in test situation

Socio-economic level of community

Curriculum (text)

Teacher use of a curriculum

Dependent Variables

Mathematics performance

Attitudes { Own ability, Mathematics }

III. *Methodology*

A. Tests Used at the Beginning and End of Kindergarten: Format and Procedure

The initial test battery given in September of the kindergarten year was planned as an evaluation of readiness for learning mathematical concepts. The tests given in May were planned to assess gain over the school year. Both tests were individually administered. The decision to develop and use individually administered tests had been made during the previous pilot testing period,[1] after reviewing published group-administered tests of mathematics achievement available for kindergarten and grade one.

Both the beginning and end-of-year batteries

[1] Two SMSG publications report the work of the test development and pilot phase of the present research. These are: Leiderman, Gloria F., Chinn, W. G., and Dunkley, M. E., *SMSG Reports No. 2*, The Special Curriculum Project: Pilot Program on Mathematics Learning of Culturally Disadvantaged Primary School Children. Stanford University, 1966; and Chinn, W. G. and Summerfield, Jeanette O., *SMSG Reports No. 4*, The Special Curriculum Project: 1965–1966. Stanford University, 1967.

were developed to minimize possible differential between disadvantaged (lower income) and more advantaged (middle income) children in handling the test situation and materials. The tests were devised so that the children responded, in most tasks, to concrete materials. When printed drawings were employed as test materials, they were used as parallel forms to those tests utilizing concrete objects. Verbal directions given by the tester were brief, simple statements, and verbal responses were necessary in few of the test items. For those requiring the children to make a verbal response, a single word or short phrase was sufficient.

Although each test took about forty minutes to administer, both the task and the materials varied frequently during this period of time. Thus, the requirement of a long attention span for good performance was considerably reduced.

1. Cognitive Process Measures

We assume that abstract concepts develop partly through awareness of certain regularities of events and, later, the categorization of these regularities. At the level of the five-year-old, such physical attributes of objects as size, shape, and color are used to systematize and categorize a wide range of perceptual experiences. Certain tasks within the beginning and end-of-year tests were used as indices of the children's awareness of and ability to label and categorize. In addition, certain mediating processes appear necessary for symbol manipulation. When the child is able to manipulate words and visual images which represent objects or experiences, rather than having to have the concrete object in his presence, then he is better able to use his experiences for more abstract symbolic thinking.

The following are the tests used to measure cognitive processes:

Labelling and Categorization Tests	*Mediating Process Tests*
Color	Vocabulary
Matching	Visual Memory
Naming	
Identifying	
Classifying	
Ordering	
Geometric Shapes	
Matching	
Naming	
Identifying	

2. Mathematics Achievement Measures

The separation of those tests identified as "Cognitive Process Variables" from those included as "Mathematics Achievement Measures" was somewhat arbitrary; the two are clearly not disjoint sets. The major criterion for calling a specific task a measure of cognitive process was that, in addition to being a learned relationship or understanding, its presence indicates a certain level of cognitive development.

Those tasks considered Mathematics Achievement Measures are:

- Counting
  - Objects
  - Members of a set
- Forming equivalent sets
- Numeral
  - Writing
  - Identification
- Ordinal number

B. Achievement Testing: Grades One through Three

Mathematics achievement is being tested at the end of each school year from grades 1 through 3. The tests measure new concepts and operations taught during each year and, as well, concepts taught in previous years which are expanded and explored more thoroughly in a succeeding year.

The measurement of achievement will gradually move from individually administered to group administered tests as the children are better able to handle printed materials as well as to comprehend and attend to verbal directions in a group situation.

C. Testing of Intelligence

The Wechsler Pre-school and Primary Scale of Intelligence was administered individually during the kindergarten year to provide a measure of each child's current intellectual function as one predictor of mathematics achievement. This test contains both verbal and performance scales and allows calculation of separate IQ's as well as a full-scale IQ.

Additional intelligence testing is planned for the following years of the Elementary Mathematics Study along with the assessment of mathematical learning.

IV. *Results*

The results presented are necessarily brief and reduced to summary statements. More detailed reports of the statistical analyses for the first two years of the study are planned for publication later this year.

A. Fall

1. Student Test Performance: Cognitive Processes

Matching of colors and shapes were simple tasks for almost all of the children studied. Identifying either colors or shapes was more difficult than matching but easier than naming them. Few of the children knew names for the four two-dimensional geometric shapes included. The circular shape was the easiest with 44 percent of the children able to name it correctly.

As a measure of classifying, the children were asked to select all of the circular shapes, for example, from a group of shapes including triangles, rectangles, and squares as well as circles. About 75 percent of the children were able to select appropriately on the above example. When, however, they were required to sort by two attributes, i.e., shape and color, the task was considerably more difficult. Even more difficult was ordering a set of shapes by size which was accomplished by only 32 percent of the children.

Performance on many of the words included within the vocabulary test was excellent. The most difficult of the twenty words and phrases included were "as many as" and "fewer than."

Visual memory was measured by having the children recall a removed object from a set of familiar objects presented. Between 78 and 96 percent of the children were able to name the removed object, depending upon the specific item. Additional analyses of the data have shown the children's performance on this test to be unrelated to their achievement on any of the other tests.

2. Student Test Performance: Mathematics Achievement

Counting of sets of objects containing three, four, or five members was accomplished by more than one-half of the children tested. Counting members of a given set, a task that utilized pictures on cards as the set members, was somewhat more difficult than counting objects.

The three achievement tasks most difficult for the majority of beginning kindergarten children were writing numerals, forming sets equivalent to a presented set, and ordinal number. Findings from the two counting tests and forming equivalent sets are consistent in demonstrating that the task becomes increasingly difficult for this age

group as the children have to deal with numbe larger than five.

3. Socio-economic Group Differences

At the beginning of the kindergarten year, th children from the lower socio-economic grou performed significantly less well than did th children from the middle socio-economic group on each of the tests, cognitive process as well a mathematics achievement, with the one exceptio of visual memory on which there was no differ ence in performance.

Behavior in the test situation was evaluated b ratings on two scales assigned by the testers im mediately after the test was administered. Th behaviors rated were attention to task and re sponse to verbal directions. Most of the childre were assigned high ratings on attentiveness t task, and no difference was found between th lower and middle socio-economic groups on thi rating scale. On response to verbal directions however, children in the middle group were rate significantly higher.

B. Spring

1. Student Test Performance

The spring test findings will be presented as changes over the school year rather than in the form of a profile of the children's performance on each test as was done for the fall results.

Significant increases in mean test scores were found for the entire sample over the school year. For each of the cognitive measures as well as for each of the mathematics achievement measures, the mean scores were significantly higher at the end of the school year than at the beginning. Thus, it appears that an important amount of learning has taken place over the course of the kindergarten year.

2. Socio-economic Group Differences

Although significant gains were made by the total population of children studied, the pattern of differences between socio-economic groups found at the beginning of the kindergarten year was repeated at the end of the year. The middle group was still significantly higher than the lower socio-economic group on all test scores, except for visual memory, as they had been in the fall. What is crucial to point out, however, is the differential pattern of gain over the school year.

Comparison of Gain Scores for the Two Socio-Economic Groups over the Kindergarten Year

Greater Gain* Shown by Lower Socio-economic Group	Greater Gain* Shown by Middle Socio-economic Group	No Difference between Gain Scores
Identification of Geometric Shapes	Ordering Geometric Shapes	Naming Geometric Shapes
Counting Buttons	Writing Numerals	Classifying Vocabulary
Counting Pictured Sets	Ordinal Number	Visual Memory-Objects
Identification of Numerals		Visual Memory-Pictures
Equivalent Sets		

* t is significant at the .05 level, two-sided test.

The above table shows the tests on which each socio-economic group made greater gains and those tests on which the difference between gains of the two groups from fall to spring was not statistically different. As was noted earlier, the lower socio-economic group performed less well than did the middle group on all of the tests at the beginning of the school year, but this lower group made significantly greater gains over the year on five of the tests, while the middle group made greater gains on three. In addition, the result showing no difference between the two groups' gain scores on five of the tests (last column in above table), provides further support to an interpretation that a structured mathematics program in kindergarten may narrow the differences in achievement of the lower and middle socio-economic groups.

On the behavior ratings made by the testers, the children looked very similar at the end of the year to the way they appeared to the testers early in

the fall. The means were somewhat higher in th spring on both response to verbal directions ar attention to task. The pattern of difference four in the fall, with the two socio-economic grou being rated very similarly on attention to task an with the middle socio-economic group rated si nificantly higher than the lower group on respon to verbal directions, was maintained.

V. *Summary and Discussion*

This report has included a description of th Elementary Mathematics Study, its purpose, d sign, kindergarten testing procedures, and som kindergarten year results. The findings presente showed a significant gain in mathematics perforn ance for the entire sample over the kindergarte year. The pattern of gain was different, howevei for the two broad socio-economic groups.

During the following years of the four-yea study, some predictions will be made and analyse undertaken to better evaluate the influences c such variables as the curriculum taught, children intelligence test scores, and their attitudes towar school on mathematics achievement. If the factor making for successful progress in mathematic during the primary years can be isolated, ther the probability of successful progress in mathe matics during the intermediate and junior higl school years can be increased.

It is hoped that this longitudinal study wil contribute to mathematics education in two ways first, in developing a theory of the interrelationships of the many factors influencing children's learning of mathematics; and second, in developing more effective materials and programs for teaching primary mathematics.

A PROGRESS REPORT on

An Experiment with Junior High School Very Low Achievers in Mathematics — Second Year

This experiment resulted from a Conference on Mathematics Education for Below Average Achievers sponsored by SMSG in Chicago, Illinois, on April 10 and 11, 1964, with financial support from the Cooperative Research Branch of the U.S. Office of Education.

An ad hoc SMSG committee met in May of 1964 to review recommendations from the Conference. This committee urged SMSG to undertake certain specific activities, one of which was the preparation of experimental materials for the low achieving junior high school pupil.

An exploratory study was conducted during the 1965–1966 and 1966–1967 school years. The encouraging results of the first year of the exploratory study led to a decision to try the material and methods developed during that year with a larger number of classes during the academic year 1966–1967. A report on the first year of this larger experiment is now in print.*

The main experiment continued with the same classes in the 1967–1968 school year. In September of the 1967–1968 school year, pupils in each experimental class were randomly separated into two groups and re-tested. One group was tested using the SAT Intermediate II tests and the other group was tested using the SAT Advanced tests. This procedure was followed in order to determine the amount of regression which took place over the summer vacation. In addition, it allowed for comparisons to be made which would indicate whether pupils had advanced far enough mathematically so that an advanced test could be used as an indicator of their achievement level. The pupils in the control classes were not tested at this time.

The following table lists the mean scores in computation and application for both experimental groups for the spring and fall testing. Group 1 is composed of those pupils in the experimental classes who were tested using SAT Intermediate II tests in both the spring and fall testings. Group 2 is composed of the pupils in the experimental classes who were tested in the spring using SAT Intermediate II tests and in the fall the SAT Advanced tests.

* For information regarding the first year of this main experiment, see Newsletter No. 29.

	Spring 1967		Fall 1967		Change	
	Comp.	Appl.	Comp.	Appl.	Comp.	Appl.
Group 1	5.7	5.8	5.1	6.3	−0.6	+0.5
Group 2	5.9	6.0	5.2	6.8	−0.7	+0.8

As can be seen from the above table, pupils in both groups regressed on computation over the summer but at the same time evidenced a substantial growth on applications. It was also apparent that the Advanced tests for junior high school pupils would discriminate as well as the Intermediate tests and therefore were used for the 1968 spring testing. Pupils were also tested using SMSG's Attitude and Mathematics Inventories.

In order to facilitate the transition from junior high school to senior high school, a major objective during the 1967–1968 school year was to move the pupils back to a more traditional classroom environment. Materials written during the summer of 1967 and seminars held with the teachers of the experimental classes were directed toward this objective. Feedback from participating teachers indicate that the change took place effectively.

Results of this two year experiment will be described in an SMSG Report which will be available in 1969.

Progress Report on a New Secondary School Mathematics Project

In 1966, upon the direction of its Advisory Board, the School Mathematics Study Group initiated the development of a new, sequential program in mathematics for grades seven through twelve.

Briefly the objectives of the project are:

(1) to provide a mathematics program for the intelligent, responsible future citizen regardless of occupation,

(2) to deal with current mathematical developments that have arisen as major forces in the many new applications of mathematics as well as in the development of mathematics itself,

(3) to include meaningful applications of mathematics while continuing to emphasize understanding and appreciation of the spirit and structure of mathematics,

and (4) to provide a program which takes into account the increased competence of the practicing teacher and the improved mathematical background of the entering seventh grade student.

During the summers of 1966 and 1967, a group of mathematicians and secondary school teachers wrote detailed outlines of the proposed program for grades seven through nine and discussed alternative proposals for curricula for grades ten through twelve. These outlines were based on the recommendations of a panel of university mathematicians, applied mathematicians, and practicing secondary school teachers, which had met in the spring of 1966.

A small group of writers prepared experimental versions of fourteen chapters during the 1966–67 academic year based on the detailed outlines. Parts of several chapters were written after the materials had been tried in seventh grade classrooms. During the 1967–68 school year these experimental chapters were tried out in sixteen seventh grade classes by sixteen teachers in five different schools. Most of the classes finished between ten and twelve of the proposed chapters. In general, the reactions of the students, the parents, and the teachers were quite favorable. Teacher and student evaluations have been systematically collected and these evaluations served as a basis for the revision of nine of the chapters during the summer of 1968.

During the academic year 1967–68 another group of writers prepared experimental versions of fourteen more chapters. These chapters are being tried out by the same group of students and teachers in the eighth grade during the 1968–69 school year.

The same group of teachers and another group of seventh grade students are trying out preliminary versions of the first twelve chapters during the 1968–69 school year. It is planned that each group of students will be involved in the program for at least three years.

Throughout the trial period participating teachers have attended biweekly seminars in which the materials and teaching problems were discussed.

The following are chapter headings of materials already written:

Experimental Version

1. The Structure of Space
2. Graphing
3. Functions
4. Informal Algorithms and Flow Charts

4M. Applications of Mathematics and Mathematical Models

5. Rational Numbers
6. Structure
7. Equations and Inequalities
8. Congruence
9. Number Theory
10. Measure
11. Probability
12. Parallelism
13. Similarity
14. Real Numbers
15. Perpendicularity
16. Coordinate Systems—Distance
17. Coordinate Geometry
18. Problem Analysis
19. Rigid Motions and Coordinates
20. Squares and Rectangles
21. Square Roots and Real Numbers
22. Approximations
23. Solution Sets of Mathematical Sentences
24. Quadratic Functions
25. Statistics
26. Systems of Mathematical Sentences
27. Parallels and Perpendiculars
28. Measurement

Preliminary Version

1. Structuring Space
2. Functions
3. Informal Algorithms and Flow Charts
4. Applications and Mathematical Models
5. The Integers and Their Structure
6. Rational Numbers
7. Equations and Inequalities
8. Congruence
9. Measure
10. Probability
11. Number Theory
12. Parallelism
13. Similarity
14. Real Numbers

Because of the experimental nature of this project the first version of these chapters was not released for general inspection. However, single copies of Chapters 1, 2, 3, 4 and 10 of the preliminary version will be made available, with commentaries, for individual inspection in April 1969. This version is not in final form and further revisions are anticipated as a result of classroom testing and evaluation.

Report on The National Longitudinal Study of Mathematical Abilities

The National Longitudinal Study of Mathematical Abilities was a five-year study, carried out during the years of 1962–1967. Over 112,000 students from 1,500 schools in 40 states participated in the Study. The primary purpose of NLSMA was to identify and measure variables associated with the development of mathematical abilities (e.g., textbook effect, attitude, teacher background, etc.).

A large population of students at each of three grade levels was tested in the fall and spring of each year, beginning with grades 4 (X Population), 7 (Y Population), and 10 (Z Population) in the fall of 1962. The X-Population and Y-Population were tested for five years. The Z-Population was tested for three years and then followed with questionnaires after graduating from high school. The design stressed three features: (1) the long-term study of a group of students—up to five years, (2) study of the same grade level at different times —for instance, grades 7–8 in 1962–64 for the Y-Population and again in 1965–67 for the X-Population, and (3) extensive data on mathematics achievement and psychological variables for grades 4 through 12.

The first analyses utilizing this extensive data are in process. The variable selected for investigation in these first analyses is the textbook. The analyses are concerned with the relationship of the textbook to mathematics achievement. Hence, the effectiveness of various textbooks, both conventional and modern, is being compared. The comparisons are carried out between groups of students defined by the textbook series used and the comparisons are made on the basis of performance on measures of mathematics achievement.

The measures of mathematics achievement administered to the NLSMA students in each test battery were not the usual end-of-year achievement tests. Certain specific mathematical topics were chosen, and scales (homogeneous sets of test items) were written to measure achievement in each topic. It was rarely the case that the particular scales included in a test battery covered all the topics normally taught in the corresponding school year.

For purposes of identifying and discussing patterns of mathematics achievement, each scale was classified according to its area of content and according to its complexity or cognitive level. The figure below shows the content areas and cognitive levels.

	Number	Geometry	Algebra
Computation			
Comprehension			
Applications			
Analysis			

Areas of Mathematics
Achievement

Most of the scales administered to students in grades 4 through 8 were classified in the content area of number systems. The emphasis in the testing of grade 9 and grade 10 students was upon algebra and geometry, respectively. In grades 11 and 12, both algebra scales and geometry scales were given emphasis. In each of the test batteries, however, scales were distributed across the classification matrix to some extent and many scales were administered more than once during the five-year period.

The central statistical procedure used in the comparison of textbook groups is multivariate analysis of covariance. A multivariate method was chosen in order to facilitate the comparison of groups on a relatively large number of achievement scales. The analysis of covariance technique was chosen in order to compensate for initial differences between the textbook groups on measures of general mental ability and previous mathematics achievement. The need to adjust the data for initial differences arose because the students available for the textbook comparison analyses were not random samples. (The assignment of textbooks to students was not controlled by NLSMA, but instead it was a matter of local or state educational policy.)

Because of the tendency of school districts to change textbook series at seventh grade and at ninth grade and because of the great diversity of textbooks used at the secondary level, it was not possible to identify any sizable groups in which the students had used the same sequence of textbooks for the entire five-year period. Consequently, no analyses are being carried out which compare groups over all five years. However, it was possible to identify several sizable groups in grades 4, 5, and 6 in which a single publisher's

series had been used by each group. Hence, the groups are being compared for the first thr years of the Study. It is possible to compare oth groups on achievement for two years in grades and 8. On the other hand, no groups are coı pared on more than one year's achievement in tı later grades.

The publication schedule for the reports ‹ the textbook comparison analyses is given belo as well as a brief description of the contents ‹ each report.

Available now.

NLSMA Report No. 1 (Parts A and B): X-Po ulation Test Batteries

NLSMA Report No. 2 (Parts A and B): Y-Po ulation Test Batteries

NLSMA Report No. 3: Z-Population Test Ba teries

These volumes contain the bulk of the tes items administered to the various populations i the Study. Only a few easily obtainable standard ized tests, which had been administered as part of various test batteries, are not reproduced be cause of copyright considerations. Both mathe matical achievement and psychological test item are included.

NLSMA Report No. 4: Description and Statis tical Properties of X-Population Scales

NLSMA Report No. 5: Description and Statis tical Properties of Y-Population Scales

NLSMA Report No. 6: Description and Statis tical Properties of Z-Population Scales

These volumes contain a brief description of each scale, a sample item, and comprehensive statistical information for each scale based upon a five percent stratified random sample of the associated population. Each test item is cross-referenced to its location in the first three NLSMA Reports. Both mathematics scales and psychological scales are included.

NLSMA Report No. 9: Non-Test Data

This volume contains the non-test data collected on the schools, communities, teachers, and families of students who participated in the Study. The volume serves to identify the variables which were formed from the non-test data and it is a comprehensive description of the NLSMA population in contexts other than mathematics achievement and psychological characteristics.

Available by late summer.

NLSMA Report No. 7: Test Development

This volume will contain a full description of the rationale for each of the NLSMA tests and of the procedures used in constructing them.

NLSMA Report No. 10: Patterns of Mathematics Achievement in Grades 4, 5, and 6: A Comparison of Six Textbook Groups in the X-Population

NLSMA Report No. 12: Patterns of Mathematics Achievement in Grades 7 and 8: A Comparison of Eight Textbook Groups in the Y-Population

NLSMA Report No. 13: Patterns of Mathematics Achievement in Grade 9: A Comparison of Eight Textbook Groups in the Y-Population

NLSMA Report No. 16: Patterns of Mathematics Achievement in Grade 10: A Comparison of Four Textbook Groups in the Z-Population

NLSMA Report No. 17: Patterns of Mathematics Achievement in Grade 11: A Comparison of Eight Textbook Groups in the Z-Population

These volumes will contain the results of the textbook comparison analyses for the first years of the Study.

Available in the Fall of 1969.

NLSMA Report No. 11: Patterns of Mathematics Achievement in Grades 7 and 8: A Comparison of Eight Textbook Groups in the X-Population

NLSMA Report No. 14: Patterns of Mathematics Achievement in Grade 10: A Comparison of Twelve Textbook Groups in the Y-Population

NLSMA Report No. 15: Patterns of Mathematics Achievement in Grade 11: A Comparison of Eight Textbook Groups in the Y-Population

NLSMA Report No. 18: Patterns of Mathematics Achievement in Grade 12: A Comparison of Eight Textbook Groups in the Z-Population

These volumes will contain the results of the textbook comparison analyses for the later years of the Study.

Available in the future.

NLSMA Report No. 8: Statistical Procedures

This volume will contain a discussion of the statistical procedures used in the textbook comparison analyses. It is a more detailed, more mathematical, and more rigorous explanation of these procedures than will be found in NLSMA Reports 10–18.

NEW PUBLICATIONS

REPRINT SERIES

The five new pamphlets listed below complete this series. Each of these pamphlets is devoted to a particular topic in mathematics and contains reprints of articles selected from a variety of journals.

- RS-11 Memorable Personalities in Mathematics: Nineteenth Century
- RS-12 Memorable Personalities in Mathematics: Twentieth Century
- RS-13 Finite Geometry
- RS-14 Infinity
- RS-15 Geometry, Measurement and Experience

SUPPLEMENTARY AND ENRICHMENT SERIES

The final two pamphlets of this series are now available. These are both designed for independent study or enrichment.

- SP-28 Order and The Real Numbers: A Guided Tour
- SP-29 The Mathematics of Trees and Other Graphs

CALCULUS

This new textbook interweaves most of the content of the earlier SMSG text *Elementary Functions* with an introduction to both differential and integral calculus. It is designed explicitly for students who plan to take the Calculus AB examination of the College Board Advanced Placement Program.

The preliminary edition of this text, together with a teacher's commentary, is now available for inspection. A revised edition will be made available for classroom use at the end of the summer of 1969.

Calculus of Elementary Functions

Chapter titles

1. Polynomial Functions
2. The Derivative of a Polynomial Function
3. Circular Functions
4. Derivatives of Circular Functions
5. Exponential and Related Functions
6. Derivatives of Exponential and Related Functions
7. Area and the Integral
8. Differentiation Theory and Technique
9. Integration Theory and Technique
10. Simple Differential Equations

Appendices

1. Functions and their Representations
2. Polynomials
3. Mathematical Induction
4. Further Techniques of Integration
5. The Integral for Monotone Functions
6. Inequalities and Limits
7. Continuity Theorem
8. More About Integrals

Secondary School Mathematics Project — Sample Chapters

This volume contains preliminary versions of five chapters, the first four and the tenth in the sequence as presently conceived. The first four chapters were taught in experimental seventh-grade classes in 1968-69 and are revisions of earlier drafts taught by the same teachers in 1967-68. Chapter 10 is a slightly revised version of an earlier draft taught by the same teachers in eighth-grade classes early in the 1968-69 school year. Further revision of these chapters is anticipated.

The Sample Chapters in this volume illustrate a number of aspects of this curriculum project: association of ideas of number and space through coordinate geometry; early introduction of the function concept; development of flow charts and algorithms as an introduction to the role and use of computers in modern society; attention to the role of mathematical models for physical situations; and introduction of concepts of probability.

PUZZLE PROBLEMS AND GAMES

This addition to the *Studies in Mathematics* series is the final report on the Puzzle Problems and Games Project sponsored by the School Mathematics Study Group. The impetus for the project came from the conviction of a number of mathematicians and mathematics educators that games and puzzles can provide a particularly effective means for developing mathematical understanding and skills. However, not every mathematical recreation has significant educational value. Hence, a first objective of the project was to survey the literature on mathematical recreations for those games and puzzles which would be appropriate for use in the mathematics classroom. This was carried out at a two-week conference held at Stanford University in June 1965. Seven mathematicians with particular interest in educational applications of mathematical recreations participated in this conference. In addition to a survey of the literature, several brainstorming sessions were held with the aim of developing new ideas for mathematical games and puzzles. The report of the conference, consisting of 27 working papers, is incorporated as Appendix D of this report.

A second objective of the project was to prepare some experimental materials of the games and puzzles which were self-contained and which would not require special training sessions in order to be used effectively by teachers. In order to implement this second phase of the project, four teams, each consisting of a mathematician and an educator, were organized. Each team was charged with the task of developing appropriate written material from one or two of the more promising ideas collected in the first phase of the project. In June 1967, the four teams met for a one-week tryout and evaluation session. Twelve teachers with their respective classes volunteered for the experiment. Each class was assigned one of the topics developed by the writing teams. In order to insure adequate preparation time, the teachers received the written material several days prior to the scheduled class sessions. Each teacher presented the material to the class during a regularly scheduled class period with the members of two or more of the writing teams acting as observers. On the basis of these observations the written material was evaluated and, in some cases, appropriate modifications were made. Appendix C contains the modified drafts of the material prepared by the four teams.

Studies in Mathematics, Volume XVIII, Puzzle Problems and Games Project Final Report

SOVIET STUDIES IN THE PSYCHOLOGY OF LEARNING AND TEACHING MATHEMATICS

Even with the recent growth of exchange programs between the United States and the Soviet Union, mathematics educators in this country have had little firsthand information about the ideas and activities of their Russian counterparts. To provide American mathematics teachers and researchers with such information, the School Mathematics Study Group and the Survey of Recent East European Mathematical Literature at the University of Chicago are collaborating on a project to translate and disseminate selections from the Soviet literature on the psychology of learning and teaching mathematics.

The first two volumes in this new series are now available. It is expected that about a dozen additional volumes will be prepared in the near future.

Each volume in the series contains one or more articles under a general heading, such as the learning of mathematical concepts, the structure of mathematical abilities, or methods of teaching mathematics. The articles form neither a random nor even a representative sample of the entire Soviet literature. Instead, the editors have chosen, from publications available to the Survey, articles that illustrate some of the most interesting aspects of recent Soviet pedagogical theory and research.

Volume I. The Learning of Mathematical Concepts

- On the Pathology of Computational Operations
  A. R. Luriya
- Operating with Concepts When Solving Geometry Problems
  V. I. Zykova
- The Psychology of Mastering Concepts: Fundamental Problems and Methods of Research
  N. A. Menchinskaya
- The Psychology of Sixth-Grade Pupils' Mastery of Geometric Concepts
  V. I. Zykova
- The Formation of Elementary Mathematical Notions
  P. Ya. Gal'perin and L. S. Georgiev

Volume II. The Structure of Mathematical Abilities

- An Investigation of Mathematical Abilities in Schoolchildren
  V. A. Krutetskii
- An Analysis of the Individual Structure of Mathematical Abilities in Schoolchildren
  V. A. Krutetskii
- An Experimental Analysis of Pupils' Mathematical Abilities
  V. A. Krutetskii
- Mathematical Aptitudes
  V. A. Krutetskii

INVESTIGATIONS IN MATHEMATICS EDUCATION

A JOURNAL OF ABSTRACTS AND ANNOTATIONS

The Advisory Board of the School Mathematics Study Group believes that knowledge of the results of research in mathematics education can be helpful and should be used in the development of programs for the improvement of mathematics education. The purpose of this journal is to make such knowledge more readily available to all those involved in SMSG activities.

This journal will contain abstracts of published research reports dealing with mathematics education. Each abstract includes an objective indication of the (1) purpose, (2) rationale, (3) research design and procedure, (4) findings, and (5) the investigator's interpretation of the findings—insofar as the information has been included in the research report.

In addition, each abstractor is given an opportunity to comment upon or raise questions about the research report for which he prepares an abstract.

The first issue of this journal contains abstracts of 16 research reports published during the first half of 1968. No fixed schedule has yet been set for later issues, but it is hoped that two issues can be prepared each year.

No subscriptions to this journal can be accepted. However, a mailing list will be maintained at SMSG Headquarters and each person on the list will be notified by postcard whenever a new volume of this journal is published. Requests for inclusion on this mailing list should be addressed to:

SMSG, Cedar Hall
Stanford University
Stanford, Calif. 94305

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SCHOOL
MATHEMATICS
STUDY GROUP

Newsletter No. 35

November 1971

STATUS REPORTS
RECENT PUBLICATIONS

The last report to the mathematical community (SMSG Newsletter No. 30) on the status of various SMSG activities was issued about two and one half years ago. In that report, three major projects and a number of smaller, but no less important, projects were discussed.

During these two and a half years, substantial progress has been made in each of these projects. Detailed reports on two of them, preparation of a new secondary school mathematics program and preparation of materials for very low achieving junior high school students, will appear in another Newsletter in the fall of 1971. Brief reports on the other activities appear below.

However, while much was accomplished during these past two and a half years, it is also true that SMSG did not undertake, during that period, any new activities. In fact, the SMSG Advisory Board has decided that, when present SMSG projects are completed, SMSG will have done the job for which it was created and that it will therefore be disbanded. It is expected that all present SMSG projects will have been completed by the end of the summer of 1972.

More on this will be found starting on page 11.

I. STATUS OF CURRENT PROJECTS

LONGITUDINAL STUDY OF ELEMENTARY SCHOOL MATHEMATICS

The Elementary Mathematics Project (ELMA) was a four-year longitudinal study carried out during the years 1966-1970. The primary purpose of the study was to assess students' progress in learning particular mathematical ideas during the beginning school years.

The participating students entered kindergarten in fall, 1966 and completed third grade in spring, 1970. The students were enrolled in selected schools in two large cities. These schools drew on residential areas which could be described as either lower or middle income groups. One lower and one middle income group in each city used the SMSG curriculum and the other comparable groups used the SRA curriculum. Of the 2000 students who entered the study in kindergarten, approximately one-half remained at the end of grade 3.

The students were tested in the fall and spring of each year. The format of the tests gradually moved from individually administered, object-oriented tests to group administered, printed tests.

In addition to tests measuring mathematics achievement and retention, standardized intelligence tests were administered during the first three years of the study, attitude scales were administered in grades 2 and 3, and the results of standardized tests administered by the school systems were obtained and included in the analyses.

The data collection for ELMA was completed in May, 1970, and analyses comparing curriculum and socio-economic groups have been carried out during the past year. The results of these analyses and a description of the design and sample are available in the following reports:

ELMA Report No. 1, *A Longitudinal Study of Mathematical Achievement in the Primary School Years: Description of Design, Sample, and Factor Analyses of Tests.*

ELMA Report No. 2, *A Longitudinal Study of Mathematical Achievement in the Primary School Years: Curriculum and Socio-Economic Comparisons and Predictions from Previous Achievement.*

Single copies may be obtained by a postcard request to SMSG, Cedar Hall, Stanford University, Stanford, California 94305.

In addition to these two reports, the following technical reports are available:

ELMA Technical Report No. 1, *Kindergarten Test Batteries, Description and Statistical Properties of Scales.*

ELMA Technical Report No. 2, *Grade 1 Test Batteries, Description and Statistical Properties of Scales.*

ELMA Technical Report No. 3, *Grade 2 Test Batteries, Description and Statistical Properties of Scales.*

ELMA Technical Report No. 4, *Grade 3 Test Batteries, Description and Statistical Properties of Scales.*

Each of these technical reports contain the ELMA test batteries administered during the year, a description of the scales from these test batteries, and comprehensive statistical information for each scale.

Available from: A. C. Vroman, Inc.
2085 E. Foothill Blvd.
Pasadena, Calif. 91109

NEW MATHEMATICAL LIBRARY

This consists of a series of short expository monographs on various mathematical subjects. The objectives of this series are the dissemination of good mathematics in the form of elementary topics not usually covered in the school curriculum, the awakening of interest among gifted students, and the presentation of mathematics as a meaningful human activity.

The authors of these monographs are mathematicians interested and well versed in the fields they treat.

Twenty-three volumes of the New Mathematical Library have been published and have been very favorably received. Two more volumes are being edited, and one other manuscript has been contracted for. Over two thirds of a million copies of these volumes have been sold to date.

This series will, of course, be kept in print by the publisher, Random House, as long as there is interest in it. Active solicitation of manuscripts by SMSG has ended, but the Executive Editor, Dr. Anneli Lax (Dept. of Mathematics, New York University) will remain in communication with Random House, which has agreed to keep open the possibility of publishing additional volumes, as an unsubsidized venture, should suitable manscripts appear.

SMSG Newsletter No. 21 (1965) contained a reference guide to the New Mathematical Library, outlining the first sixteen volumes and suggesting appropriate grade levels for each chapter. It is planned to bring this reference guide up to date, but in the meantime, single copies of Newsletter No. 21 may be obtained by a postcard request to SMSG, Cedar Hall, Stanford University, Stanford, California 94305.

NATIONAL LONGITUDINAL STUDY OF MATHEMATICAL ABILITIES

A previous progress report on the National Longitudinal Study of Mathematical Abilities (NLSMA) appeared in SMSG Newsletter No. 30, March, 1969. The analysis, summarization, and description of the massive set of data gathered for NLSMA is continuing, with many analyses completed or in progress. A series of NLSMA Reports is being published. A Report to each participating school has been made, a summary Report is nearing completion, and a NLSMA Data Bank has been established to enable the larger mathematics education community to make use of the data.

The Study

The major purposes of the Study were (1) an assessment of how successful new mathematics curricula have been in reaching their goals; (2) an investigation of variables related to mathematics achievement and mathematical abilities, regardless of curricular approach, and (3) to provide criteria for evaluating mathematics programs.

A large population of students at each of three grade levels was tested in the fall and spring of each year, beginning with grades 4 (X-Population), 7 (Y-Population), and 10 (Z-Population in the fall, 1962. The X-Population and Y-Population were tested for five years. The Z-Population was tested for three years and then followed with a questionnaire after graduation from high school. The design stressed three features: (1) the long-term study of a group of students—up to five years; (2) study of the same grade level at different times—for instance, grades 7–8 in 1962–64 for the Y-Population and again in 1965–67 for the X-Population; and (3) extensive data on mathematics achievement for grades 4 through 12.

The data in NLSMA were gathered from three major sources: students, teachers, and school districts. The students supplied most of the information, chiefly through their performance on the tests. Mathematics achievement was measured in each of the spring test sessions and in three of the fall sessions. Most of the achievement measures were short scales designed to measure specific components of achievement.

Various psychological instruments were used in NLSMA to gather information about the students' abilities, attitudes, and personalities. Questionnaires were sent to the teachers to learn something of their attitudes and experience, and to the school districts to obtain descriptive information about the students, the schools, and the communities.

Background Documents

The first nine NLSMA Reports, 11 volumes, were designed to provide source material on the Study. *NLSMA Report Nos. 1, 2,* and *3* contain most of the test batteries for the X-, Y-, and Z-Populations, respectively. Only a few standardized tests are omitted due to copyright restrictions. *NLSMA Report Nos. 4, 5,* and *6,* for the X-, Y-, and Z-Populations, respectively, contain a brief description of each scale, a sample item, and comprehensive statistical information for each scale.

NLSMA Report No. 7 describes the development of mathematics tests, the selection and adaptation of psychological tests, the development of a conceptual model for viewing mathematics achievement and the work of writing teams and conferences called for various purposes. *NLSMA Report No. 8,* under preparation, will provide information on statistical procedures and computer programs used in the data management and data analyses for the Study. *NLSMA Report No. 9* summarizes the nontest data collected from schools, communities, teachers, and families of students who participated in the Study. The volume is a comprehensive description, or source document, of the NLSMA Populations in contexts other than mathematics achievement and psychological characteristics.

Patterns of Mathematics Achievement

The first analyses of NLSMA data were comparisons of groups, determined by the textbook used, to describe the patterns of mathematics achievement associated with each textbook. Textbook groups were compared on a range of achievement criteria (from 5 to 27 variables) including measures at the cognitive levels of computation, comprehension, application, and analysis.

Nine NLSMA Reports have been prepared, and six published to date, for these analyses. *NLSMA Report No. 10* was for patterns of mathematics achievement in grades 4, 5, and 6. Patterns of mathematics achievement in grades 7 and 8 were treated in *NLSMA Report Nos. 11* (X-Population) and *12* (Y-Population). *NLSMA Report Nos. 13, 14,* and *15* were for the Y-Population grade 9 algebra, grade 10 geometry, and grade 11 algebra or intermediate mathematics respectively. *NLSMA Report Nos. 16, 17,* and *18* were for the Z-Population, grades 10, 11, and 12. The Reports that have been published are available through Vromans, Inc.; the others will be finished soon.

Correlates of Mathematics Achievement

A second major set of data analyses was concerned with the correlates of mathematics achievement. Variables examined as potential correlates included affective measures such as attitude, anxiety, and self-concept specifically oriented to mathematics; cognitive process measures such as spatial visualization, reasoning, verbal ability, induction, deduction, or speed of logical reasoning; measures of school and community characteristics; measures of teacher background and of teacher attitudes; and teacher assigned grades in mathematics, science, social studies, English, and reading.

Approximately 90 potential correlates were examined for some 25 mathematics achievement measures in each population. The analyses were repeated on four different samples: boys using SMSG textbooks, boys using conventional textbooks, girls using SMSG textbooks, and girls using conventional textbooks.

These analyses were designed to examine (1) the relationship between the correlate and the mathematics achievement (e.g., Is there a relationship? If so, is it nonlinear? Is it positive? Negative?), and (2) whether any relationship found was the same for high, medium, and low ability students. Summary data and graphs were computer generated and reproduced for these analyses. The reports are therefore resource documents containing a wealth of hypotheses for further examination in theory and in practice; detailed interpretations of the relationships were not attempted.

NLSMA Report Nos. 21 through *26* are for these analyses. Respectively, the Reports contain the results for the affective (attitude and role inventory) variables, for the cognitive variables, for the teacher variables, for the school and community variables, and finally, a summary.

Other Analyses

Under- and Over-achievers

NLSMA Report No. 19 reported an investigation of under- and over-achievers in mathematics for grades 4, 5, and 6. Regression techniques were used to identify samples of under-achievers and over-achievers. Then variables from the NLSMA data were used to characterize each sample and to distinguish between them.

Correlates of Attitude toward Mathematics

NLSMA Report No. 20, under preparation, will report several analyses that examine the use of attitude as an outcome variable for mathematics instruction. Some analyses have been done separately for modern and conventional textbook groups; other analyses contrast the two.

The Prediction of Mathematics Achievement

NLSMA Report No. 27, in preparation, will present analyses designed to determine an efficient set of predictors, using regression techniques, of mathematics achievement. A stratified random sample of each NLSMA population was used. Summary data and tables have been presented, along with a description of the analyses, but with a minimum of interpretation.

Teacher Effectiveness in Mathematics Instruction

NLSMA Report No. 28, also in preparation, presents an analysis of teacher effectiveness utilizing residual gain scores obtained with regression techniques.

Affective Variables and Mathematics Achievement

NLSMA Report No. 29, in preparation, will contain background information on the development of the affective (e.g., attitude, anxiety, self-concept) instruments used in NLSMA, together with some analyses of relationships between these psychological and mathematics achievements.

Follow-up Study of the NLSMA Z-Population

The Z-Population students were each sent a follow-up questionnaire at the end of the first year after completing twelfth grade. *NLSMA Report No. 30* will present a summary of this follow-up data and some analyses that have used the data.

School Reports

Each school that participated in NLSMA was provided a detailed analysis of the results of the school's mathematics program, in terms of student progress, at the end of the Study. These individual school reports were strictly confidential and sent only to the test center director in the particular school. A Schools Reports Manual, describing the school reports and providing background information, was prepared to assist school personnel with the reading and interpretation of their own School Report.

NLSMA Data Bank

The NLSMA Data Bank is now available for educational researchers to use. This data resource contains information on 112,000 students from 1,500 schools in 40 states.

Basic policy on use of this data bank has been set by the SMSG Committee on NLSMA Data.* Of primary importance is the SMSG agreement not to release any information which could be ascribed to a particular individual or school. Any request for data will be carefully screened from this point of view, and any data provided will

* E. G. Begle, Stanford University; L. R. Carry, University of Texas; J. Kilpatrick, Teachers' College, Columbia University; J. F. Weaver, University of Wisconsin; and J. W. Wilson, University of Georgia.

have been stripped of identifying information. In general, data will be provided only for a random sample of the population.

Each request for NLSMA data, after being screened as indicated above, will next be reviewed to make sure that the data is actually available in the amounts requested. In addition, an estimate will be made of the cost of extracting the requested data from the data bank. Cost estimates will be prepared as carefully as possible, but cannot be guaranteed to be completely accurate.

This estimate will be returned to the researcher. No data will be sent out until the researcher agrees to pay the entire cost of preparing it. In addition, the researcher will agree to provide SMSG with a copy of any report of his analysis of the data.

Each request for NLSMA data should consist of: a) a statement of the problem which the researcher is investigating, b) a careful delineation of the particular sample of NLSMA students and a list of the particular variables for which data is requested, and c) an outline of the statistical analyses to be carried out. Different researchers asking for similar data for similar purposes will be brought to each other's attention.

Requests for analyses of data to be carried out by the SMSG staff will be considered but, because of limited staff time, will not often be possible.

RESEARCH IN MATHEMATICS EDUCATION

In order to keep the SMSG Advisory Board, and its committees, informed about current research in mathematics education, four issues of a Journal of Abstracts and Annotations have been published and distributed within the SMSG organization. A fifth volume will be published in the Spring of 1972. This publication has also been made available to the public and has attracted wide interest.

Because of this wide interest, negotiations are under way to have the responsibility for continuing this journal transfered to an appropriate organization.

While the main research-related activity at SMSG headquarters has the analysis of data collected in the National Longitudinal Study of Mathematical Abilities and the Elementary Mathematics Project, one research project was carried out during 1970 and 1971. This was aimed at obtaining empirical evidence on the degree to which teachers' understanding of algebra affects the algebra achievement of their students in the beginning ninth grade algebra course. Over three hundred teachers took two algebra tests in the summer of 1970 while they were attending NSF summer institutes. Their students were tested at the beginning and again at the end of the 1970–71 school year. The data is now being analyzed and a report will be prepared.

The SMSG Panel on Research met twice during the 1968–69 academic year. As a result of the discussions during these meetings, it was decided to try to construct one or more theoretical models of mathematics learning which could be used as a course of suggestions for research studies in mathematics education. It was also decided to make available descriptions of presently existing evaluation models and to develop new models for new purposes.

Some pilot work in both these directions was carried out subsequently. The Panel will hold a meeting during this academic year to draft a final report.

II. PLANS FOR THE FUTURE

RESPONSIBILITIES FOR SCHOOL MATHEMATICS IN THE 70's

A report of a conference on this topic has been prepared. The following proposal, extracted from this report, was approved by the SMSG Advisory Board at its January 1971 meeting and has been forwarded to the Conference Board of the Mathematical Sciences:

PROPOSAL FOR A NEW ORGANIZATION FOR MATHEMATICS EDUCATION

On October 23 and 24, 1970, SMSG sponsored a Conference on Responsibilities for Mathematics Education in the 70's. The extensive proceedings of this conference have been examined by an ad hoc committee of the SMSG Advisory Board.

At its working session in Washington on December 11 and 12, the ad hoc committee organized ideas suggested by the conference into seven major categories—objectives, teacher training, research, curriculum, evaluation, communication, and exploiting the work of the past decade in the next decade. Each of the attached summaries points up the major problems in the given category with indications of what might and should be undertaken in the 70's. For some of the most urgent problems, specific projects are suggested for action.

The ad hoc committee thought of who might be encouraged to take the action. At least one-half of the suggestions would easily fit into SMSG as presently constituted. One or two could be incorporated into the present activities of CBMS. Several are new kinds of activities and new ideas, e.g., coordinated research efforts and the consortium on teacher training, requiring a different kind of organization.

Overall, there is a recurring feeling—implicit and explicit—that the nature and size of the problems identified and the actions suggested require the participation of a wide variety of people in the mathematics community. Many of the problems are too big to be undertaken by a single university, school, or other existing organization. Marshalling the efforts of the mathematics community at large requires some SMSG-type organization that can cut across the various specialties needed to work on the problems.

We believe that the organization set up by SMSG was appropriate to deal with the problems of the past decade and that a number of current problems could be attacked by the present organization. However, we feel that a fruitful attack on the problems of the 70's ideally requires new people, fresh ideas, and new organizations.

We believe that one new organization is needed to plan, to stimulate, and to coordinate work on the problems identified. The organization itself should initiate action. Action is needed if the organization is to have vitality and by its vitality attract competent people with needed expertise. Furthermore, productive action results in the confidence and acceptance necessary to attract and keep widespread support of the academic community as well as financial support from government or foundation funds. However, the organization should be free to enlist the cooperation of schools, universities, and other groups in its various activities.

The organization should consist of a Director, some permanent staff, and a working Board of Directors of from five to seven members. Board members should meet three or four times a year for sessions of three or four days so that they can be aware in depth of the activities of the organization and can provide thoughtful leadership. The Board should be representative of the various constituencies in the mathematics community. Since the effectiveness of the Board and the Director will depend very much on the quality of the people, special effort should be made to ensure

the appointment only of individuals of sound judgment and with a wide understanding of mathematics education.

The Conference Board of the Mathematical Science seems a natural parent for such an organization because CBMS does represent all organizations. Procedures for election of the Director and the Board would have to be worked out with CBMS.

We recommend that the SMSG Advisory Board go on record as supporting the formation of an organization as described herein. The problems identified in our recent conferences would provide an initial focus for the organization. It would, of course, be encouraged to identify other problems, initiate planning, stimulate the mathematics community, and move to some course of action.

We also recommend that action on this recommendation take place as soon as feasible so that the organization will be functioning at the time SMSG activities are completed. The problems in mathematics education are crucial and serious. They deserve forthright action by the best talent of the country.

Ad Hoc Committee
E. G. Begle
Burton Colvin
Donovan Johnson
Karl Kalman
Jeremy Kilpatrick
Joseph Payne
Henry Pollak

January 5, 1971

AVAILABILITY OF SMSG PUBLICATIONS

Many SMSG publications will continue to be available until 1977, five years after the end of SMSG activities. At that time the Conference Board of the Mathematical Sciences will be asked to review then existing SMSG publications and to recommend which, if any, should be kept in print any longer.

During the present academic year, the demand pattern for each SMSG publication will be reviewed and a decision will be made as to whether the publication should be withdrawn, having served its purpose, or be allowed to go out of print when the current stock is exhausted, or be kept available by reprinting when necessary. These decisions will be announced in an SMSG Newsletter early in 1972.

III. NEW PUBLICATIONS

Since the appearance of Newsletter No. 34, March, 1971, a number of new publications have been added to the SMSG list. Single copies of the following can be obtained by a postcard request to: SMSG, Cedar Hall, Stanford University, Stanford, California 94305.

ELMA Report No. 1, A Longitudinal Study of Mathematical Achievement in the Primary School Years: Description of Design, Sample, and Factor Analyses of Tests.

ELMA Report No. 2, A Longitudinal Study of Mathematical Achievement in the Primary School Years: Curriculum and Socio-Economic Comparisons and Predictions from Previous Achievement.

The following new publications may be obtained from Vroman's. An order form appears on page 17.

SOVIET STUDIES IN THE PSYCHOLOGY OF LEARNING AND TEACHING MATHEMATICS

Volume V — The Development of Spatial Abilities

INVESTIGATIONS IN MATHEMATICS EDUCATION

A Journal of Abstracts and Annotations
Volume 4

NLSMA REPORTS

No. 15 Patterns of Mathematics Achievement in Grade 11: Y-Population

No. 19 The Non-Intellective Correlates of Over- and Under-achievement in Grades 4 and 6.

ELMA TECHNICAL REPORTS

No. 1 Kindergarten Test Batteries, Description and Statistical Properties of Scales

No. 2 Grade 1 Test Batteries, Description and Statistical Properties of Scales

No. 3 Grade 2 Test Batteries, Description and Statistical Properties of Scales

No. 4 Grade 3 Test Batteries, Description and Statistical Properties of Scales

ORDER INFORMATION

1. Accredited schools will be given an educational discount of 30%. Shipping charges will be billed to the purchaser.

2. Other institutions or individuals interested in mathematics education will be given an educational discount of 30% on orders totaling $10 or more. Shipping charges will be billed to the purchaser.

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9. As it takes longer to process an order during the rush period in July, August, and September, we urgently suggest that you place your order well in advance of your needs. Orders will be shipped on a first come, first served basis.

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	CONFERENCE REPORTS		
______	A Conference on Responsibilities for School Mathematics in the 70's (CR-9)	$ .75	______

SCHOOL
MATHEMATICS
STUDY GROUP

NEWSLETTER NO. 39

August 1972

Final Report of the SMSG Panel on Research

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PREFACE

The SMSG Panel on Research met twice during the 1968-69 academic year. At the first meeting the Panel reviewed the state of research in mathematics education and formulated a number of recommendations. These were reviewed and approved by the SMSG Advisory Board.

The second meeting was devoted to a discussion of ways of implementing two of the recommendations. One of these was:

To construct one or more theoretical models of mathematics learning and teaching and to make them available to the mathematical community as a source of suggestions for research in mathematics education.

This report is an incomplete response to this recommendation. It is incomplete since no theoretical model of mathematics teaching and learning is provided. However, research programs are suggested which, if carried out, should eventually provide enough empirical information to suggest one or more usable theories.

A preliminary draft of this report was reviewed by the Panel at its final meeting in November, 1971. This final draft of the report incorporates suggestions made by the Panel.

A careful review of the recent (and some of the older) literature disclosed no ready-made theory appropriate to the current mathematics curriculum. However, this review did reveal a substantial number of empirical studies of mathematics learning. [Reasons for confining attention to empirical studies will be found in Begle (1969).] Unfortunately, despite the number of such studies, no clear pattern of findings is apparent.

In most of the studies the number of subjects is too small to encourage generalizations. Almost never is it possible to compare two of these studies, since they use different treatments, different instruments, and different kinds of subjects.

Despite these drawbacks, we cite these studies in this report whenever they are at all relevant since they often provide arguments for (or against) further specific research projects.

The research programs detailed in Chapters III and IV are designed to overcome, to some extent, the disadvantages mentioned above of the research which has been carried out so far.

The first two chapters provide some background information. They are presented in more detail than most mathematics educators need for the benefit of our colleagues in other parts of the educational community who may wish to know what we are planning.

Let it not be thought that we are suggesting that no mathematics education research be undertaken that does not fit into the programs we describe. On the contrary, we encourage fresh ideas and fresh attacks in the basic problems facing us. Nevertheless, we feel that unless a concerted effort is made to work along the lines we suggest, it is unlikely that we will ever accumulate the knowledge base we need to formulate useful theoretical models of mathematics learning.

Finally, we wish to emphasize that in this report we are concerned only with the teaching and learning of mathematics. We have nothing to say about problem solving, despite the fact that probably our most important overall goal in mathematics education is to develop in each student the ability to apply the mathematics he has learned to real world problems. A recent review by Kilpatrick (1969) makes it clear that our empirical information about problem solving is so scarce and disconnected that attempts to theorize about it now would be futile.

Chapter I
MATHEMATICS AND ITS STRUCTURE

A prerequisite to a study of the learning of mathematics is a clear understanding of the nature of the mathematics to be learned. We consider mathematics to be a set of *interrelated, abstract, symbolic systems*. In order to make clear the meaning of the preceding sentence we will try to explicate each of the italicized words.

Let us first consider the part of mathematics dealing with the whole numbers. The basic *symbols* used are:

$0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, \times, \div, =, <$, and $>$.

A *system* is based on these, the elements of the system being certain combinations or "strings" of these basic symbols. There are familiar grammatical rules which tell us that the following are grammatically correct phrases and hence elements of the system:

$$207, \quad 121 + 2, \quad 12 \times 12 + 6 \times 24;$$

while the following are not:

$$2t, \quad 12 \times 24 \div, \quad 2 \quad 7.$$

Also, there are other familiar grammatical rules which tell us that the following are grammatically correct sentences:

$$2 + 3 = 5, \quad 2 \div 3 = 6, \quad 181 \times 92 < 92 \div 181;$$

while these are not:

$$2 \times 3 =, \quad 2 + 3 = 6 -, \quad 181 \times 92 <.$$

Two operations, addition and multiplication, are defined next. The addition and multiplication tables:

+	0	1	2 ···	9
0	0	1	2 ···	9
1	1	2	3 ···	10
2	2	3	4 ···	11
	·	·	·	·
	·	·	·	·
	·	·	·	·
9	9	10	11 ···	18

×	0	1	2 ···	9
0	0	0	0 ···	0
1	0	1	2 ···	9
2	0	2	4 ···	18
	·	·	·	·
	·	·	·	·
	·	·	·	·
9	0	9	18 ···	81

are used to define sums and products of single digit elements of the system. For other elements, the addition and multiplication algorithms are brought in.

The operations of subtraction and division are defined in terms of addition and multiplication respectively, and algorithms for these operations are provided.

By means of these algorithms the truth or falsity of any sentence involving these four operations and the equality sign can be determined.

A similar but somewhat more involved procedure can be used to determine the truth or falsity of sentences involving the symbols $<$ and $>$. We need not go into the details of this.

An important aspect of the structure of this system is that there are some *laws* or *principles*:

(a) $N + M = M + N$
(b) $N \times M = M \times N$
(c) $(N + M) + P = N + (M + P)$
(d) $(N \times M) \times P = N \times (M \times P)$
(e) $N \times (M + P) = (N \times M) + (N \times P)$
(f) $N + 0 = N$
(g) $N \times 1 = N$
(h) $N \times 0 = 0$

which are always true, no matter what whole numbers N, M, and P are.

Still another aspect of the system is the *place value* principle, exemplified by

$$30 + 7 = 3 \times 10 + 7.$$

There are other aspects of the system of whole numbers, e.g., factoring and primes, which we need not go into at this time. There are, however, two remarks which must be made here.

A. It is possible for a student to learn the system of whole numbers, including the grammar of whole number expressions, the addition and multiplication facts, place value, the computational algorithms, and the structural principles, in a purely formal, rote fashion. (After all, we can program a computer to carry out any whole number computation or to find the solution set of any whole number open sentence.)

B. The system, as described, is *redundant*. Some aspects of the system can be deduced from others. Thus, principle (h) above can be deduced from the other principles. Similarly, the algorithm for addition, for example, can be deduced from the above principles, including the place value principle.

C. Some aspects of the system logically precede other aspects. For example, the algorithm for multiplication requires the algorithm for addition.

The system of whole numbers is not untypical of mathematical systems in general. It contains a collection of symbolic objects, in this case, whole number symbols. Certain operations are defined on these objects and certain laws state relationships between these operations. A number of different subsets of the system can be specified (i.e., set of prime numbers, set of perfect numbers), and certain laws or principles hold for these systems (i.e., there are infinitely many primes) and for the relationships between the full system and the subsystems (i.e.. $\pi(N) \cong N / \ln N$).

These aspects of the whole numbers are enough to explicate the words "abstract," "symbolic," and "system" used in the first paragraph of this chapter, and these aspects of the whole number system constitute its *structure*. However, the structure of mathematics is more than the sum of the structures of its individual systems. In order to see that this is so, and also to help to explicate the word "interrelated," let us turn to another system, that of all finite sets of concrete objects.

First we note the operation of "pairing" the elements of two disjoint sets A and B, leading, in each case, to one and only one of the relations

$$A < B, \quad A \sim B, \quad A > B.$$

It is an experimental fact that, given two sets, the relation between them is independent of the way in which the pairing operation is carried out. Thus we can think of the pairing operation as an algorithm for ascertaining which of the above relations holds between two sets.

A number of principles concerning these relations are also experimental facts, such as: when $A < B$ and $A' \sim A$, then $A' < B$, etc.

Two other operations on sets are also fundamental aspects of this system. The *union* operation associates with any two sets A and B, the set $A \cup B$ consisting of those objects, and only those, which belong to A or to B or to both. The (cartesian) product operation associates with a pair of sets A and B the set consisting of the union of disjoint copies of B—one for each object in A. This is denoted by $A \times B$.

Again, there are certain principles concerning these operations, and again these are experimental facts. For example (all sets disjoint):

1) $A \cup B \sim B \cup A$,
2) when $A \sim A'$ and $B \sim B'$, then $A \cup B \sim A' \cup B'$,
3) $A \times B \sim B \times A$,
4) when $A \sim A'$ and $B \sim B'$, then $A \times B \sim A' \times B'$.

So far we have left implicit certain aspects of the concept "set" which ought now to be mentioned. First, in the operation of pairing the members of two sets, the sizes, weights, color, density, etc., of the objects in the sets is ignored. Hence the relations $<$, $\sim$, and $>$ do not depend on these physical aspects of the objects constituting the sets.

Second, when we have specified a particular set by indicating the specific physical objects which it contains, we agree that it remains the same set no matter how the physical location of the objects may be changed.

For these reasons, we can consider this system to be an abstract, symbolic one, even though it is, of course, much more concrete than is the system of whole numbers.

It is clear that sets are familiar, tangible things which are frequently the objects of our attention in everyday life. Also, the operations of pairing and forming the union (and to a lesser extent the cartesian product) are frequently carried out by almost everyone (although the terminology may be unknown and the principles relating the operations may be unrecognized).

Consequently, it is important to note that this system of finite sets of concrete objects is closely related to the system of whole numbers and that, in fact, the latter can be *constructed* from the former. Let us review briefly how this is done.

We start by separating all sets into *equivalence classes,* where each set in any particular equivalence class is equivalent to every other set in that class and is equivalent to no set in any other class. We next attach to each equivalence class a number, i.e., an element of the symbolic system of whole numbers.

Now we can *define* the relations $<$, $=$, and $>$ between whole numbers by referring back to the relations $<$, $\sim$, and $>$ between sets chosen from the equivalence classes corresponding to the numbers. Similarly, we can *define* the operation of addition of numbers N and M by choosing a set A with N elements and a (disjoint) set B with M elements and defining $N + M$ to be the number attached to the set $A \cup B$.

The operation of multiplication is defined in an analogous way using the cartesian product instead of the union.

When we look at the situation from this point of view, we see that the concepts of "number," "addition," "multiplication," "less than," "equals," etc., connect the symbolic system of whole numbers with the system of finite sets of concrete objects.

Thus the structure of mathematics has two parts. On the one hand, each mathematical system has its own internal structure. On the other hand, there are linkages between different systems which also contribute to the structure of mathematics.

Finally, it should be noted that in this chapter we have considered, for illustrative purposes, only two mathematical systems: the whole numbers and finite sets of concrete objects. Of course there are many more systems, all linked together in one way or another. For example, before the end of elementary school the systems of one-, two-, and three-dimensional geometry are introduced, as well as the systems of non-negative rationals and the integers. In each case, the internal structure of the system is exposed, but also the linkages between systems are not only made clear but are emphasized in the pedagogical procedures favored today.

Chapter II
THE OBJECTS OF MATHEMATICS LEARNING

As a student learns the mathematical structures and the structure of mathematics discussed in the previous chapter, he is, at any given moment, faced with a particular mathematical object to learn. It can be of a number of different kinds. Since these different kinds of objects may call for different teaching-learning processes, it seems wise to distinguish them clearly.

1. *Facts*

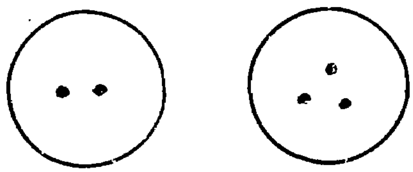
There are two kinds of mathematical facts. Some facts are arbitrary, such as "the symbol for the word 'seven' is 7." Presumably, these are learned by rote and the laws of rote learning which have been worked out by psychologists are applicable to the learning of this kind of mathematical fact.

2. *Principles*

Other mathematical facts, however, are not arbitrary since they can be deduced from other facts. We call these facts "principles." For example, $7 \times 8 = 56$ is a fact, but it can be deduced from others:

$$7 \times 8 = 7 \times 7 + 7 \times 1 = 49 + 7 = 56.$$

Similarly, the fact $2 + 3 = 5$ can be deduced from this diagram:



by remembering the definition of addition in terms of union of sets.

These two examples are quite simple, and there are many other such, but there are also many complex principles. For example, every mathematical theorem states such a mathematical fact or principle. Also, the exact format of any particular mathematical algorithm is a mathematical principle.

Presumably, this kind of fact can also be learned by rote, but the weight of the evidence is that there is a better method and that learning is improved if the fact is taught in a "meaningful" fashion, i.e., if the connections with other facts or principles are pointed out to the student. The evidence on this point will be reviewed in the next chapter.

3. *Concepts*

As an example, consider the concept of "triangle." Triangles form a subset of the set of all geometric figures. In order to determine whether a given figure is a triangle, certain aspects or "dimensions" of the figure need to be considerd: the number of sides, the straightness of the sides, etc. At the same time, it is part of the concept of triangle that certain other dimensions are irrelevant, e.g., the lengths of the sides, the size of the angles, and the orientation of the figure.

Other examples of concepts are: three, rectangular array, mathematical sentence, fractions, congruence, greater than. (Note that in the case of the last three it is a relationship between pairs of objects that the concept specifies.)

4. *Skills*

While it is theoretically possible for an individual to know and understand a considerable number of mathematical systems without being able to carry out any mathematical operations. we normally would disapprove of such a situation. One of the objectives of mathematics education is the development of a certain amount of skill (measured in terms of accuracy and speed) in carrying out certain mathematical operations. Indeed, it is probably the case that a certain amount of proficiency with mathematical operations is necessary for the learning of some parts of mathematics.

Examples of skills are: pairing the members of two sets, counting, forming the union of two sets, adding two numbers (using the addition algorithm), measuring the length of a line segment, constructing the perpendicular bisector of a line segment, multiplying both sides of an equation by the same number, representing addition of numbers by motions along a number line.

As indicated in the first paragraph of this chapter, different kinds of mathematical objects may require different kinds of teaching-learning processes. It is therefore of interest to compare the above classification of the objects of mathematics learning with a few other classification systems.

The hierarchy of learning types set forth by Gagné (1970) is one of the most elaborate and best known of these. His "signal" learning does not seem relevant to the learning of mathematics unless, perhaps, we take affective objectives into account.

His "stimulus-response" learning and "chaining" together seem to correspond better than any of his other types to the development of mathematical skills; although we shall argue later that an additional condition, beyond those that he states, is important for this kind of learning.

His "verbal association" learning probably applies to the learning of arbitrary facts. The role of "multiple discrimination" learning in mathematics education is not clear.

Gagné's view of "concept" learning is similar to ours, as will be seen in the next chapter. His "principle" learning corresponds to the learning of our principles discussed above.

A different kind of classification was used in the National Longitudinal Study of Mathematical Abilities and has been described by Romberg and Wilson (1969). This taxonomy of educational objectives in mathematics was derived from the well-known taxonomy of Bloom (1956). The NLSMA taxonomy does not include a cognitive level corresponding to the learning of arbitrary facts, since testing for knowledge of such facts was not part of the NLSMA program. An extension of this model by Wilson (1971) showed one way the knowledge category could be incorporated. The cognitive level "computation" corresponds to the mathematical skills discussed above. The level "comprehension" is appropriate to concept and principle learning, while the "application" level requires a mixture of skills, concepts and principles. The NLSMA cognitive level "analysis" is quite similar to the problem-solving process mentioned at the end of the Introduction.

In summary, the objects of mathematics learning listed above are not in conflict with either of these two classifications.

Chapter III
CANONICAL INSTRUCTIONAL PROCEDURES

We are going to outline two different research programs. One, described in the next chapter, is concerned with curriculum variables. The other, described in this chapter, is concerned with instructional variables.

We start by noting that almost all the mathematics that students learn is learned as the result of instruction. In order to study the learning of mathematics it is therefore necessary to take the instructional procedures into account. It is well known that there are many different kinds of instructional procedures and that there are many different dimensions on which they can differ.

One way of carrying out programmatic research on the learning of mathematics is to specify a particular teaching procedure in detail, so that all of the parameters of the procedure are clearly visible. These parameters can then be varied one at a time so that the effects of each one on student learning can be studied.

Although, as mentioned above, there are many different kinds of teaching procedures, the choice of the procedure to use in a research program is practically forced on us. Despite widespread interest in such procedures as discovery teaching, team teaching, mathematics laboratories, etc., the expository teaching procedure is the only one which can be specified in detail and hence is the one we must choose.

This restriction, however, is not a serious one. In the first place, we know that expository teaching is effective. Most mathematics instruction always has been, and still is, expository. Any success of our mathematics education program is thus associated with expository teaching. But even a casual inspection of any standardized test for, say, upper elementary school students, together with the associated norms, which increase as grade level goes up, is enough to indicate that our school mathematics program, and hence expository teaching, is effective to some degree.

This is not to say that expository teaching is equally effective for all students, or that no improvements are possible. In fact, the objective of research in mathematics education is to find ways of increasing the effectiveness of our mathematics program and, in particular, to make better adjustments to individual differences in students.

Nevertheless, no non-expository procedures have been demonstrated to be more effective, overall, than exposition. A report by Shulman and Keislar (1966) shows that comparisons of discovery teaching with conventional expository teaching balance out, with neither method showing a clear advantage over the other. A review by Hermann (1969) reports similar findings.

For other procedures currently of interest, empirical information is hard to find. In a review of mathematics laboratories, Fitzgerald (1970) reports not being able to find empirical evidence in favor of this method. An older study by Biggs (1967) suggests that it is less effective than traditional teaching methods. Vance and Kieren (1971) summarize the research, which indicates that laboratories are no more effective than other instructional procedures. Bowen (1969) found that the use of games was less effective than exposition. Moodey et al. (1971) found that activity oriented instruction provided no advantage.

Coop and Brown (1970) found that exposition was superior to an independent-problem-solving method of instruction.

Paige (1966) found team teaching no more effective for junior high school students than conventional instruction.

We attach the label "canonical" to the teaching procedures described below as a reminder that each one is given in sufficient detail to make all the parameters visible and to make the procedure reproducible.

A *canonical teaching procedure* for a particular mathematical skill is a sequence:

$$T = [A_1, A_2, \ldots, A_8]$$

of individual teaching acts, A_j, where

A_1 consists of a statement of the objective of T, i.e., a statement of the expected student ability after being exposed to T.

A_2 provides a name, if there is one, for the skill (e.g., "long division").

A_3 consists of a listing of immediately prerequisite skills, principles, and concepts.

A_4 consists of the *development* of the algorithm, using a particular case, from the prerequisites.

A_5 consists of $k > 0$ repetitions of A_4, differing only in using a new particular case each time.

A_6 consists of the student's development of the algorithm, using a new particular case.

A_7 consists of $h > 0$ practice items, with immediate feedback, after each one, to the student.

A_8 is an $n > 0$ item test, with immediate feedback to the student, designed to indicate how well the student has learned the skill.

Note that this teaching process calls, in act A_4, for the *development* or deduction of the algorithm. This development requires the use of concepts and principles of one kind or another. Thus, the algorithm for addition of whole numbers with two or more digits can be developed from the place-value aspects of the structure of the whole number system and the connection between addition and the union of sets (see School Mathematics Study Group (1963), pp. 67–75).

Thus this teaching process provides the learner both with the skill in question and also a new principle, the one which explains why the algorithm works. This is not to say, though, that the teaching process teaches both the skill and the understanding at the same rate.

The canonical teaching could have been defined in a different way, by replacing *development* of the algorithm with *demonstration*. In this case, we think of the student merely copying what the teacher does, and the matter of principles involved does not come up. We do not use this definition of the standard teaching process for skills because the literature suggests strongly that this kind of rote learning is less effective than the meaningful learning embodied in the definition above.

A *canonical teaching procedure* for a particular concept is a sequence:

$$T = [A_1, \ldots, A_9]$$

of teaching acts, A_j, where

A_1 states the objective of T.

A_2 consists of the provision of a name for the concept.

A_3 consists of the listing of immediately prerequisite concepts, skills, and principles.

A_4 consists of a statement of the definition of the concept.

A_5 provides $k > 0$ different examples of the concept and demonstrates that each one is indeed an example.

A_6 provides $m > 0$ different non-examples of the concept and demonstrates that each one is indeed a non-example.

A_7 provides $j > 0$ new examples of the concept and uses them to identify irrelevant dimensions.

A_8 consists of $n > 0$ practice items, with immediate feedback to the student after each one.

A_9 is a $p > 0$ item test, with immediate feedback to the student.

A *canonical teaching procedure* for a principle will also be a sequence:

$$T = [A_1, \ldots, A_7]$$

of teaching acts, A_j, where

A_1 is a statement of the objective of T.

A_2 provides a name, if there is one, for the principle.

A_3 lists immediately prerequisite skills, concepts, and principles.

A_4 consists of the deduction or demonstration of the principle, using a particular case.

A_5 consists of $k > 0$ repetitions of A_4, using different cases each time.

A_6 consists of the student's application of the principle to $m > 0$ different cases, with immediate feedback to the student after each application.

A_7 is an $n > 0$ item test, with immediate feedback to the student.

The individual acts within the blocks of these canonical teaching procedures have been chosen on the basis of findings of empirical research studies. These findings have seldom been unanimous, but even when they disagree, that is an indication that more definitive research is desirable.

Engel (1968) and Kennedy (1968) both found that making goals explicit facilitated learning. Collins (1972) also found that, for junior high school students, specific objectives facilitated learning. Piatt (1969) found that when teachers were trained in writing behavioral objectives, their students learned more. On the other hand, Amidon and Flanders (1961) and Bierden (1968) found that the setting of goals had no effect. Ikeda (1965) used teacher ratings of test items as an indirect indication of teacher-held objectives. These had low correlations with student achievement.

There have been several studies of the effects of providing verbal labels. Chan and Travers (1966) and Ward and Legant (1970) found that meaningful labels were facilitating. London and Robinson (1968) found that naming line drawings facilitated memory of them. Seidel and Rotberg (1966) found that naming rules facilitated the construction of computer programs. Hagen and Kingsley (1968) found that verbal labels facilitated short term memory.

Overing and Travers (1966) found that verbalization of knowledge acquired during training was facilitating. Radford (1966) and Gagné and Smith (1962) had similar findings. However, Palzere (1967) found that verbalization had no effect.

Keats and Duncan (1972) found that verbal definitions were more effective than numerical examples as feedback when students made errors.

Phillips (1960) found a significant correlation between arithmetic skills and technical mathematical vocabulary for elementary teachers. Curry (1966) found that middle SE students benefited from a vocabulary review. Eagle (1948) found that mathematics achievement was not correlated with reading comprehension but was with mathematics vocabulary.

Finally, there have been numerous studies indicating the importance of verbalization for concept formation in young children. Representative references are: Flavell et al. (1966), Keeney et al. (1967), Kendler and Kendler (1961), and Milgram and Noce (1968). Serra (1952) provides a review of the role of verbalization in concept development. Minke (1969) provides a useful review of different theoretical views of verbal mediation.

Statements of objectives and provision of verbal labels may be analogous to providing "advance organizers" as discussed by Ausubel (1963). In this connection, see Scandura and Wells (1967).

Aside from a report by Darnell and Bourne (1970), which indicated that pretraining on relevant prerequisite dimensions facilitated performance on a classification task, no empirical information concerning block three was found.

Block four, development or deduction, in the canonical teaching procedures for skills and for principles has been investigated many times. What is called "meaningful" teaching usually requires a considerable amount of development of mathematical ideas. The keynote speaker in the campaign for meaningful teaching was Brownell (1935), and Brownell and Moser (1949) provided one of the first convincing demonstrations of the advantages of meaningful over rote teaching. Other such experiments, though on smaller scales, were carried out by Kenney and Stockton (1958), Krich (1964), Schmidt (1965), Schrankler (1966), C. Smith (1968), Stuart (1965), and Treadway and Holister (1963).

Gray (1965), Hammond (1962), and Tallmadge (1968) all found that learning procedures which took into account the structural properties of mathematical systems were beneficial. However, Greathouse (1965) found no such advantage.

A number of studies, Hopkins (1965), Pigge (1964) and (1967), Shipp and Deer (1960), Shuster and Pigge (1965), and Zahn (1966), have shown that by increasing the percent of classroom time devoted to development (and hence decreasing the time devoted to practice) student achievement is increased.

It is generally agreed that SMSG and similar modern texts devote more time to "meaningful" presentations of mathematics than did the conventional pre-1960 texts. Student achievement in SMSG programs were found to be better than in conventional programs by Davidson and Gibney (1969), Friebel (1967), Grafft (1966), Grafft and Ruddell (1968), Hungerman (1967), McIntosh (1964), Mastain (1964), Miller (1961), Moore and Cain (1968), Osborn (1965), Payne (1963), Simmons (1965), Tryon (1967), and Ziebarth (1963).

In a very ingenious experiment, Skemp (1962) demonstrated the advantage of a kind of meaningful learning over rote learning.

Of course, not all findings have gone in the same direction and there are some studies, Clark (1966), Flanagan (1969), Hall (1967), Horn (1969), Jick (1969), Joseph (1963), Pincus (1955), Sherer (1967), Shuff (1962), Wick (1963), and Williams (1962), which report no advantages for meaningful learning.

The evidence in favor of providing a variety of examples of a concept or skill or principle is both sparse and mixed. Gagné and Staff (1965) varied the number of examples given in a programmed geometry unit. This variable had no significant effect. However, Gagné and Bassler (1963) found that on a retention test a restricted variety of examples in the learning program was deleterious. Youniss and Furth (1967) found that multiple instances had an advantage over single ones in classification of logical concepts. However, Frayer (1969) found that eight instances were no more effective than four in a geometric concept learning task.

Rector (1968) found that for low ability students it was best not to provide examples, but merely to characterize the concept. For other students, providing examples before or after the characterization had no effect.

Johnson and Stratton (1966) found that a mixture of classification, definition, providing synonyms, and using in sentences was more effective than any single one of these ways of teaching concepts.

Flook and Saggar (1968), Gilman (1969), Hillman (1970), Karraker (1967), Sassenrath and Garverick (1965), and Wittrock and Twelker (1964) all found that feedback of results on tests resulted in better student achievement than when no feedback was provided. However, both Gilbert (1956) and Sullivan et al. (1967) found that feedback was deleterious.

Johnston et al. (1969), Hillman (1970), Marshall (1969), Nystrom (1969), and Wiebe (1966) found that knowledge of results on problems or tests should be provided as soon as possible. Proger (1968) found that daily tests were more effective, for student achievement, than less frequent ones.

However, Sassenrath and Yonge (1968) found no difference between immediate and delayed information feedback for the learning of psychology, but the latter was better for retention. In another, similar study (1969) they obtained the same results. Sturges (1969) found delayed feedback better than immediate if the feedback contained enough information. More (1969) found 2½ hour and 1-day feedback delay better than 4-day or immediate feedback.

The evidence reviewed above is not, of course, sufficient to guarantee the worth of any particular act in a teaching block. However, it is sufficient to indicate that in any serious program of research on the learning of mathematics, the role of each kind of teaching act needs to be investigated.

Similarly, we make no claim of having presented a complete list of instructional variables. We do feel that of all the instructional variables that have been reported in the literature, the ones that we have listed show the greatest promise of research payoff, but we by no means wish to persuade anyone to avoid studying other variables.

At the beginning of this chapter we promised

to describe a research program concerned with instructional variables. The canonical teaching procedures which we described above are an integral part of this program.

We recommend that a moderate number (approximately a dozen) instructional units be prepared, together representing each of the varieties of canonical teaching procedures, concerned with a variety of mathematical topics, and appropriate for a spread of grade levels from elementary school to senior high school. For each such instructional unit one or more criterian tests should be provided.

We recommend that these instructional units be administered to large numbers of students who together represent a wide variety of student variables.

We recommend that premeasures of as many student knowledges, abilities, and attitudes as possible be administered in the course of the various administrations of these instructional units.

The above recommendations will require a large number of separate experiments, carried out by many experimenters in many parts of the country and with many different kinds of students. We know (Begle and Geeslin (1972)) that teachers vary widely in their effectiveness and also that their effectiveness cannot be assessed in advance.

To compare two or more administrations of the same instructional unit, the teacher variable must be bypassed. We therefore recommend that the instructional units be prepared in programmed form.*

Once the effects on student achievement of one of these instructional units is well understood, we recommend that alternate versions be prepared, each of which differs from the original with respect to only one instructional variable.

If such a research program is organized and carried out, it will be necessary to establish one or more centers each of which will gather together and synthesize the information obtained by the administration of one or more of the instructional units.

*SMSG has constructed three such programmed instructional units. The first is intended for upper elementary school students and is concerned with factoring and primes. The second is devoted to probability and is appropriate for junior high school. The third, intended for senior high school students, deals with negative number bases.

These three programs will be made available through ERIC.

This research program should, if carried out, provide us with the extent of empirical information about instructional variables, and the way they are related to student variables, needed to suggest a non-trivial theory of mathematics learning.

Chapter IV
CURRICULUM VARIABLES

There seem to be two main categories of curriculum problems that would repay careful investigation. The first category deals with the need or desirability of including particular mathematical topics in the curriculum. The second deals with alternative ways of developing particular mathematical concepts.

Very few of the topics included in the present mathematics curriculum were selected on the basis of empirical information. Many topics are being taught today just because they always have been taught. A number of new topics were introduced into the curriculum in recent years, not because they were felt to be intrinsically valuable, but rather because they were thought to facilitate the learning of other, more important topics.

These new introductions have not always paid off. Thus, for example, Kavett (1969), McCormick (1965), Schlinsog (1968), and S. Smith (1968) all found no significant value in the study of non-decimal systems. However, Lerch (1963) found a study of a non-decimal base useful, and Diedrich and Glennon (1970) found that a study of non-decimal systems led to a better understanding of decimal systems on a post-test but not on a retention test.

Nelson (1966) found that teaching estimation improved arithmetic understanding for sixth graders but not for fourth.

McAloon (1969) and Retzer (1967) both found that the study of logic led to improved student achievement in mathematics, but Phillips (1968) found that a study of formal logic did not improve ability to construct proofs in a course on field theory. Deep (1969) found that a unit on logic did contribute to ability to construct geometric proofs.

Lyda and Taylor (1964) found that a study of modular arithmetic did not lead to a better understanding of decimal numeration. Herbst (1967) found that a study of coordinate systems did not lead to better understanding of geographic concepts. H. Smith (1968) found that a study of set theory did not lead to improved achievement on percentages. Shulte (1970) found that a unit on probability and statistics for ninth grade general mathematics students did not increase computational skills.

While the topics investigated in these studies were all ones recently included in or suggested for the modern mathematics curriculum, the lack of empirical studies of more conventional topics is no guarantee that they are all as important as we once thought. Indeed, Educational Development Center (1969) has recently suggested that the amount of time devoted to arithmetic computation in the upper elementary grades is too great.

It is clear that we need to know much more than we now do about the importance of each topic now included in the mathematics curriculum, and that without such information future curriculum improvement will be difficult to bring about effectively.

To obtain this information, we would have to start with a specification of certain topics as being intrinsically important. For each of these, one or more objective tests would have to be provided and used to specify "satisfactory" performance.

Next, working backward in the curriculum, for a given intrinsically important topic, each mathematical topic which seems logically to be an immediate prerequisite would be investigated to see if it in actual practice did contribute to satisfactory performance on the important topic. Again, specific objective tests for each prerequisite topic would have to be provided. "Satisfactory" performance on the prerequisite topic would be scores on these tests which lead to satisfactory performance on the important topic.

[Actually, the situation may be more complicated. There may be a number of prerequisites for any particular topic, and a weak performance on one prerequisite might be compensated for by a more than satisfactory performance on another.]

Now, the prerequisites of each of the above prerequisites could be investigated in the same way, and so on back to the beginning of the educational process.

Before considering an outline of such a research program, let us turn to the other category of curriculum problems. At many spots in any mathematics curriculum, we are faced with the necessity of choosing between two or more ways of developing a mathematical concept or between two or more sequential orders in which concepts can be introduced.

There have been some empirical studies of such problems, but much remains to be done. In some of the research reports listed below, significant differences were found but in other cases the different approaches were equally effective.

Uprichard (1970) found that not all of the six possible sequences for the introduction of the ideas "equivalent," "greater than" and "less than" were equally easy for preschool children.

Jackson (1965) and Scrivens (1968) both compared different approaches to numeration systems.

Alternative ways of presenting the various operations with whole numbers have been studied by Burkhart (1967), Coxford (1965), Dilley (1970), Ekman (1966), Gibb (1956), Gran (1966), Gray (1965), Hervey (1966), Ho (1966), Kratzner (1971), Sandefur (1965), Schell (1964), Scott (1963), Tietz (1968), Wiles et al. (1972), and Zweng (1964).

Willson (1969) compared the two sequences, decimal-common and common-decimal, for the introduction of fractions and found no significant differences.

Different ways of developing operations with fractions were studied by Anderson (1965), Bat-Hee (1968), Bergen (1966), J. Bidwell (1968), Capps (1962) and (1963), Green (1969), Sluser (1962), and Triplett (1962).

Different approaches to the teaching of decimals were compared by O'Brien (1967), of percent by C. Bidwell (1968), May (1965), McMahan (1959), and Wynn (1965), of integers by Coltharp (1968), and of function by Nelson (1968).

Bassler (1966) and Holtan (1963) studied the effects of real-world settings of exercises.

Maletsky (1961) found no significant differences between three ways of developing topics in elementary statistics.

In geometry, Genkins (1971) found paper folding more effective than mirrors for young children while Cheatham (1969) found no difference between paper folding and ruler-and-compass procedures for junior high school students.

Earle (1964) found the use of color effective in teaching descriptive geometry. Monroe (1965) found no advantage from integrating analytic geometry with calculus. Both Pettofrezzo (1959) and Schaumberger (1962) found a vector approach to analytic geometry no more effective than a conventional one. On the other hand, Bundrick (1968) found vectors advantageous.

Dart and Hicks (1968) found that, for non-majors, the sequence logic-sets was better than sets-logic.

Caruso (1966) found that an abstract deductive approach to algebra was more effective than a concrete inductive one. But Shelton (1965) found both approaches to the teaching of the limit concept equally effective.

In order to obtain the kind and amount of information about curriculum variables that will make possible substantial improvements in school mathematics programs a widespread research effort is needed.

We recommend that every mathematics education researcher choose one problem in this general area, review whatever is already known about the problem, and prepare a detailed description of an experiment which will provide further information about the problem. Since we are interested here in curriculum rather than instructional variables, it will probably not be necessary, in general, to prepare anything as elaborate as a canonical instructional unit, but the possibility of doing so is, of course, not ruled out.

Each such experiment will need to be carried out many times and in many places. As one contribution to this program, we suggest that each faculty member engaged in training doctoral students require each of his students to carry out at least two of these experiments as part of his practical training. The details of the experimental procedure would be obtained from the researcher responsible, but the student would be free to exercise his ingenuity and indulge his own particular interests in choosing which experiment to try, in designing a battery of pretests, and in selecting the experimental subjects.

In conclusion we reiterate that if such cooperative research programs are carried out, we have a chance of making substantial progress in increasing our understanding of how mathematics is learned and how mathematics curricula can be improved.

On the other hand, if we are not willing to work cooperatively and if we continue in our individualistic fashion, then a decade from now we will be able to refer to a much larger number of reports of empirical research on instructional and curricular variables. But all those reports will, in their totality, be of no more use to us then than the ones we have listed here are today.

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